Splitting hairs

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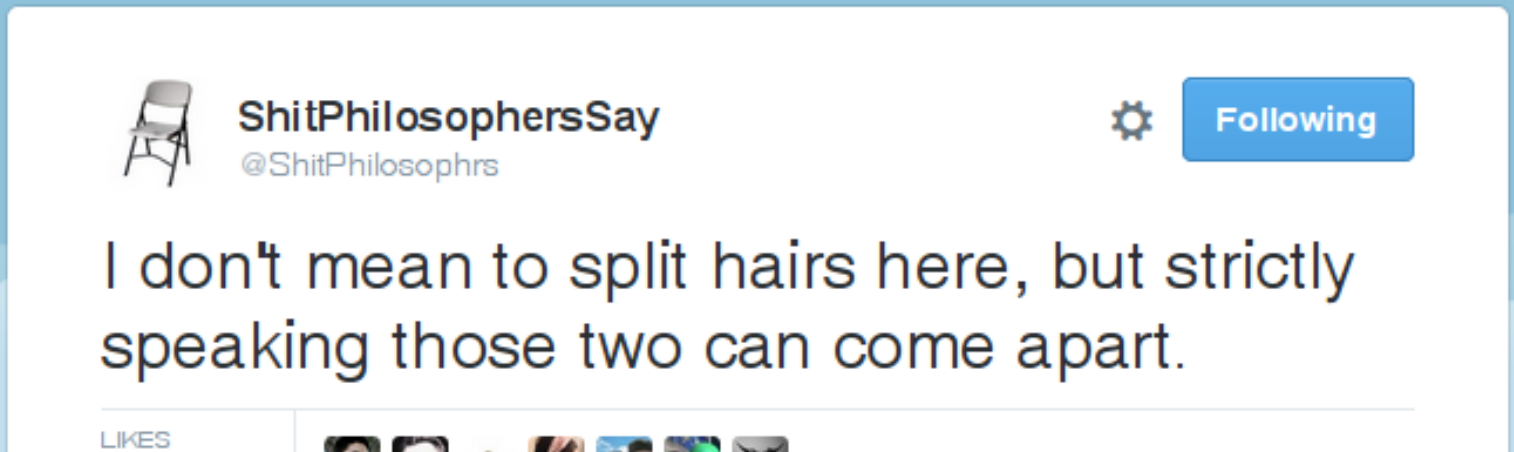
Thanks to Chris Barker, Philippe de Groote, Larry Moss, Reinhard Muskens, Valeria de Paiva, and the anonymous reviewers for many helpful comments.



There is exactly one country that is second largest in the Americas, namely the United States.

What is its population?

How many letters in ‘lava’? in English?

What are we talking about?

It depends on context, experience. Dynamic!
Regardless, we talk on.

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A discourse move to morph the domain

Not just narrowing down possibilities.

> *"A computer scientist earns brownie points for showing that two things that seem different are actually the same, whereas a linguist earns brownie points for showing that two things that seem the same are actually different."*

A useful move.

What is the value of a bound variable?

What is a country? language? grammatical construction? letter? offense? cold front? tumor? lunch? thing? How many things?

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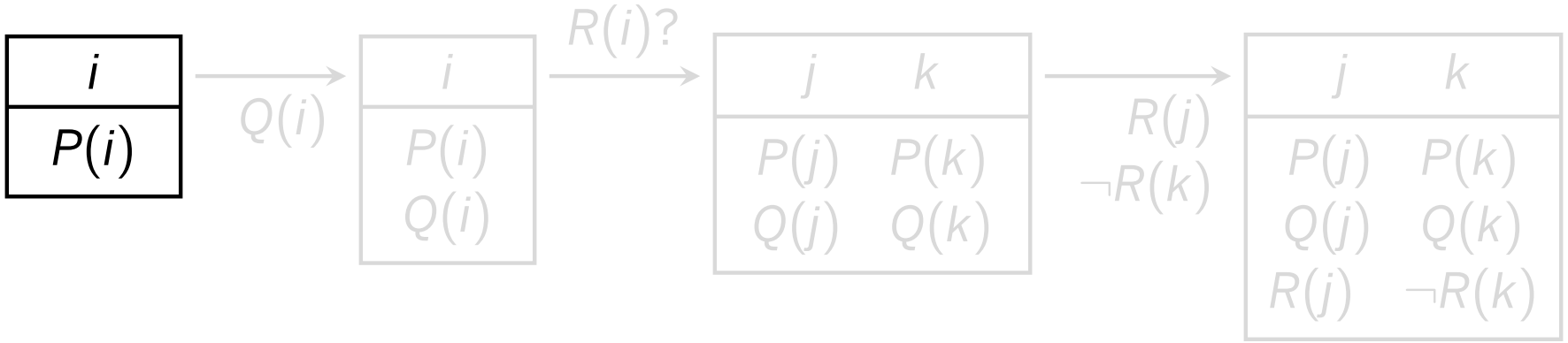

Not just a relation on possible worlds.

What kind of underspecification?

What kind of underspecified individual?

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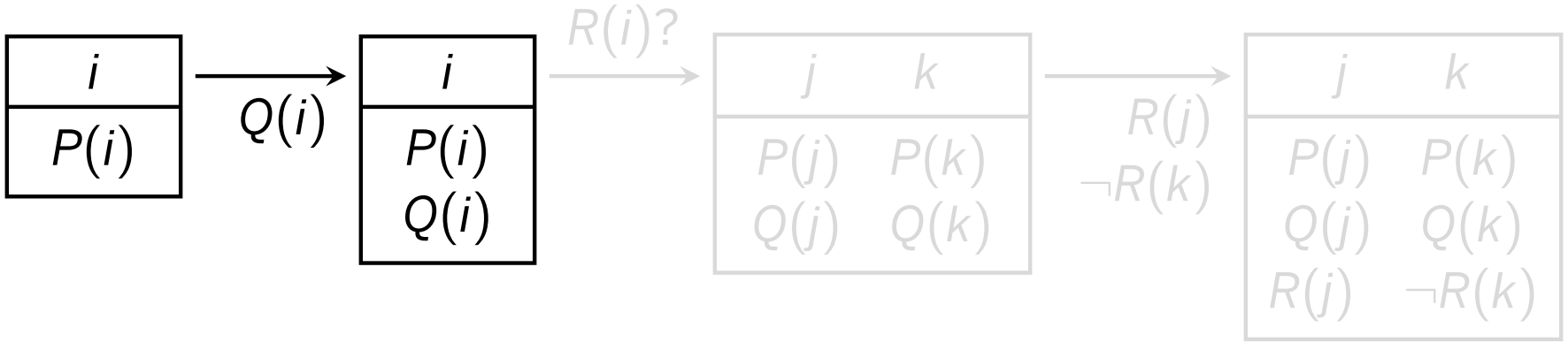

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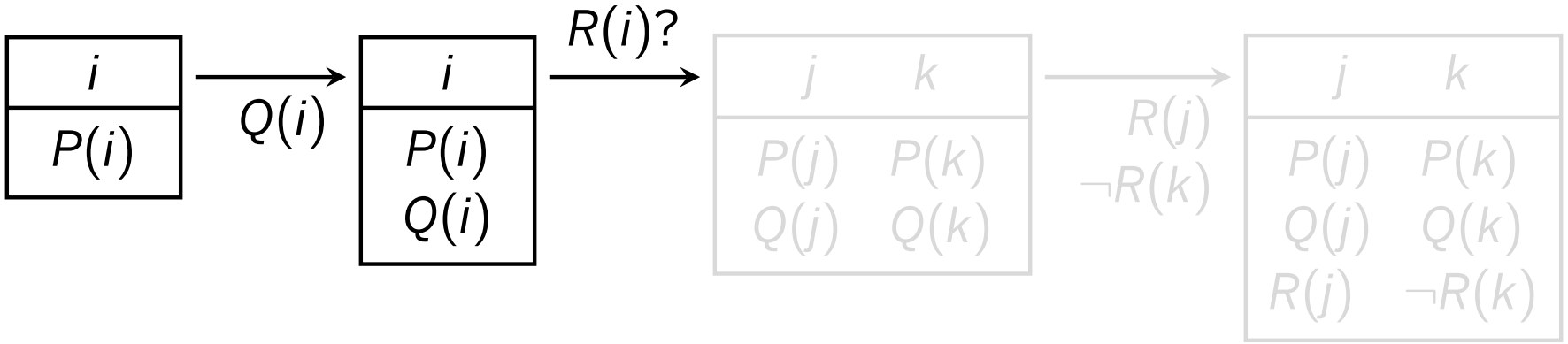

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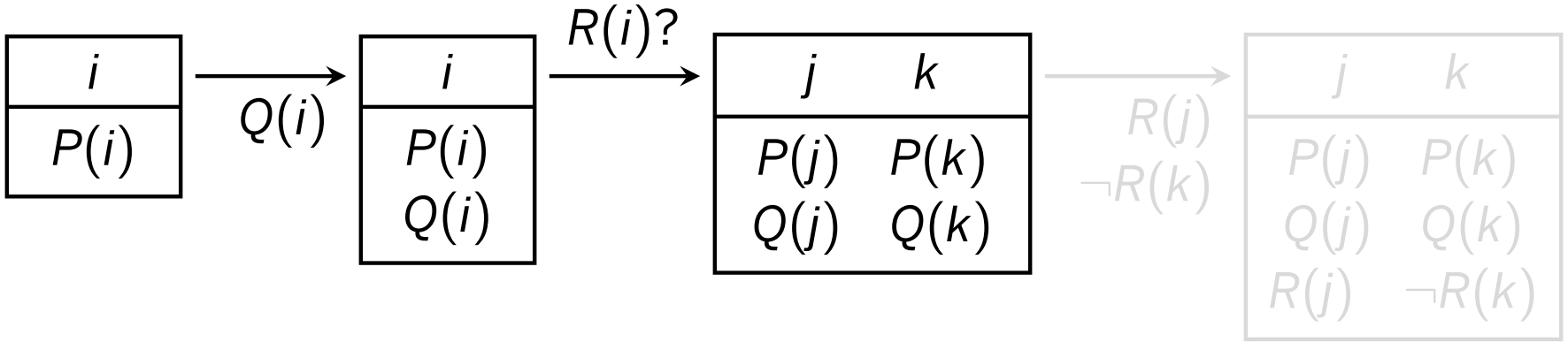

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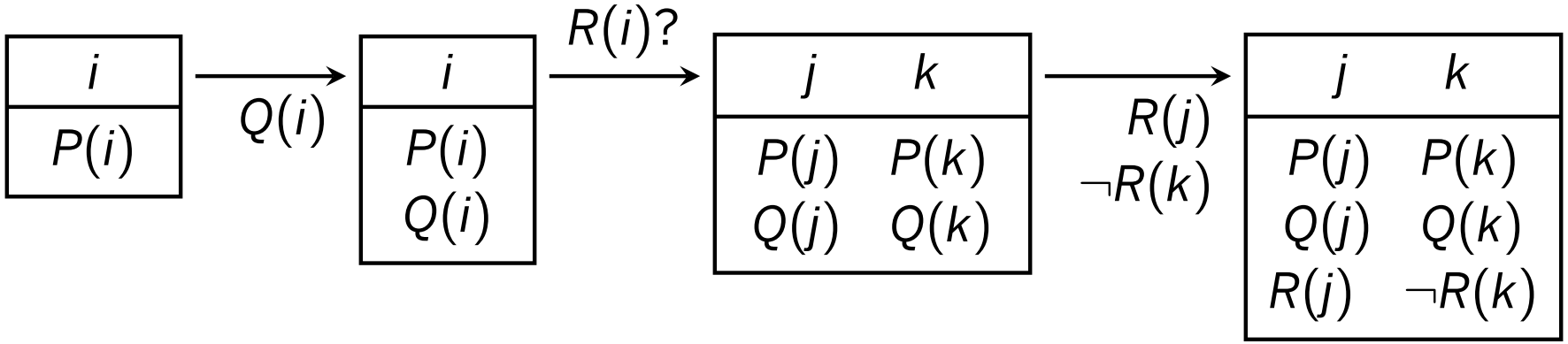

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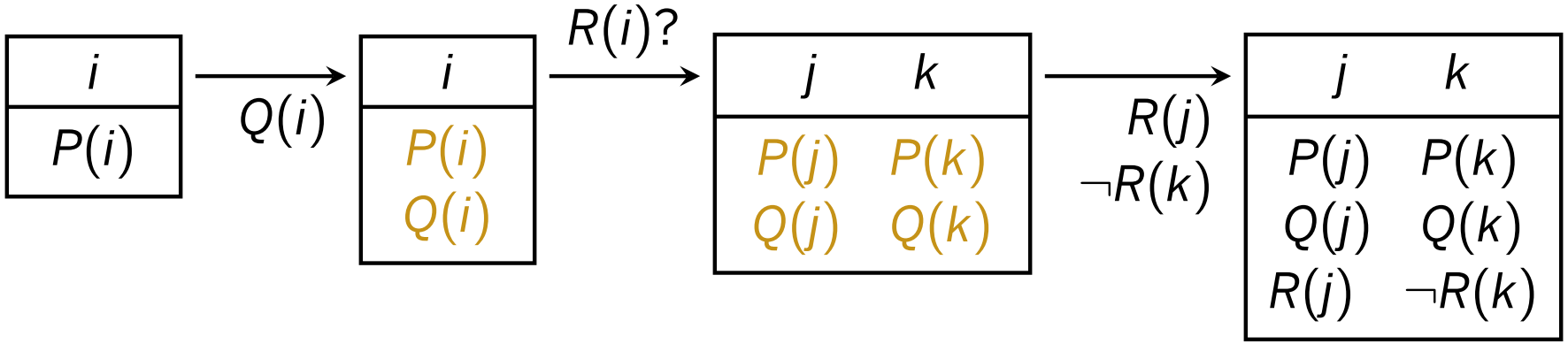

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A working hypothesis

There is no fact of the matter what things there are,
though there is a fact of the matter what stuff there is.

Between textual entailment (syntax and algorithms, not shared reality) and model-theoretic semantics (presupposed things, not stuff).

A modal logic of discourse states.

Key idea: individual accessibility relation.

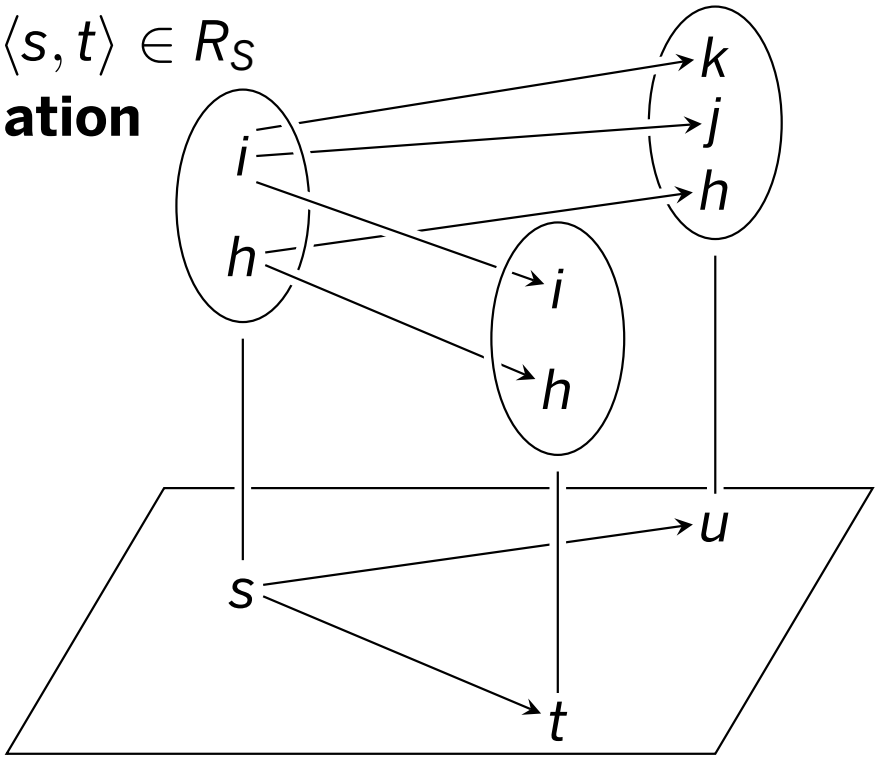
Path-lifting modal predicate logic: Frames

A **frame** $F = \langle S, R_S, D, R_D \rangle$ consists of

1. a set S of **states**;
2. a **state accessibility relation** $R_S \subseteq S \times S$;
3. a function D mapping each state $s \in S$ to a set $D(s)$, called the **domain of individuals at** s;
4. a function R_D mapping each pair $\langle s, t \rangle \in R_S$ to an **individual accessibility relation** $R_D(s,t) \subseteq D(s) \times D(t)$.

Notate accessibility infix: $s\,R\,t$, $x\,{}_sR_t\,y$

(a weird counterpart relation)



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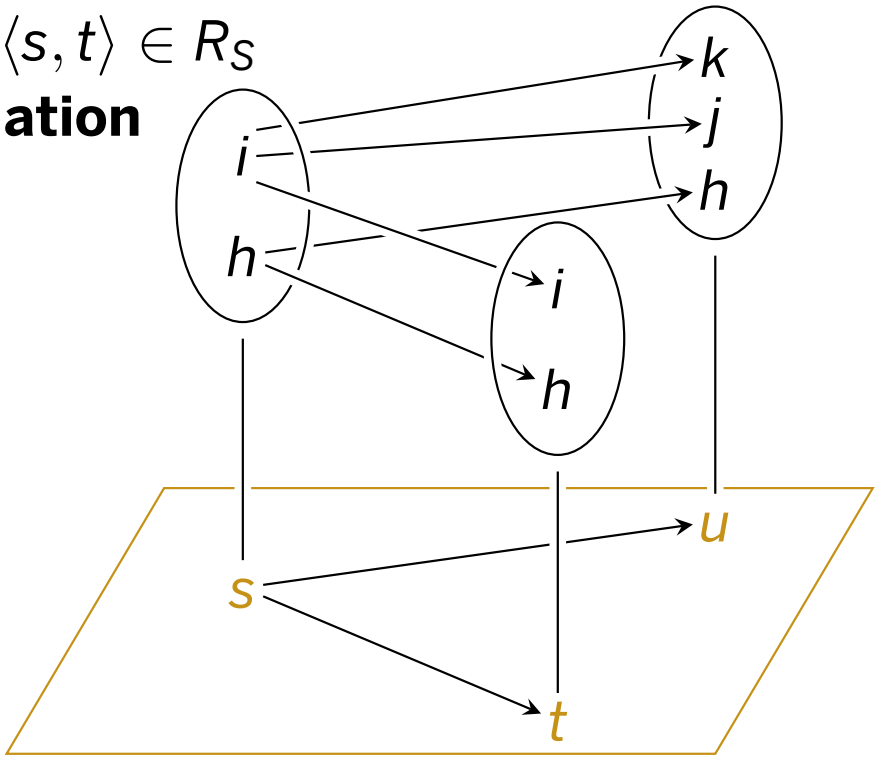

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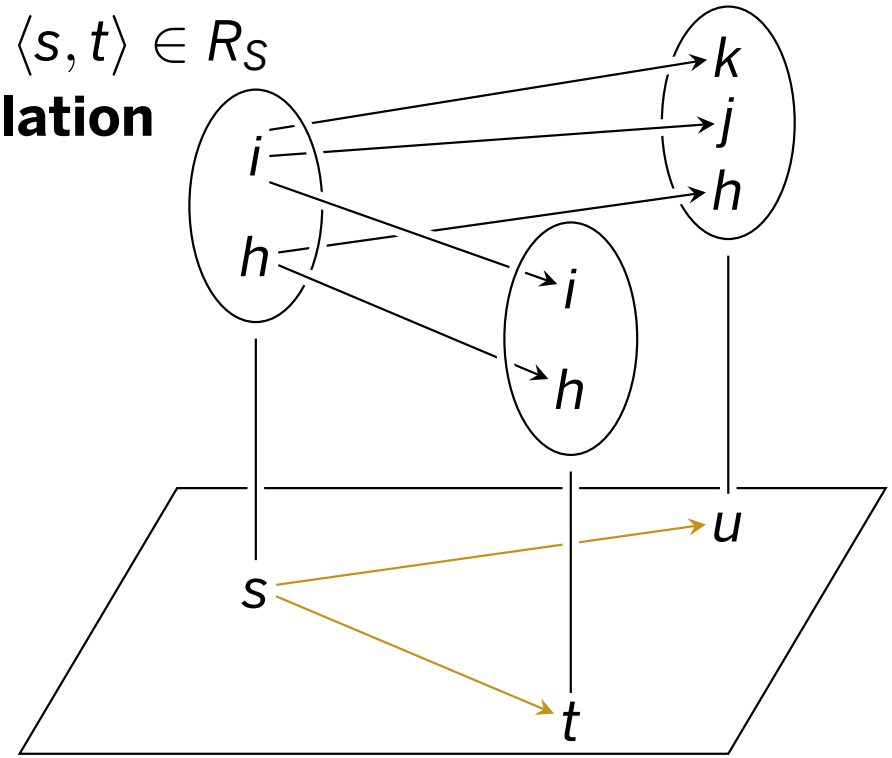

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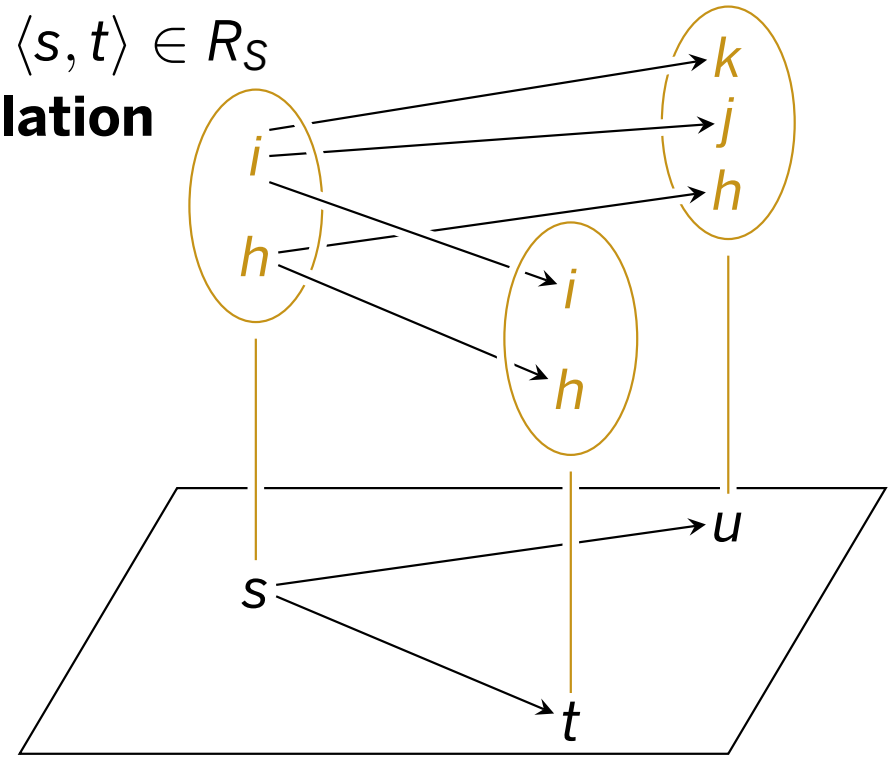

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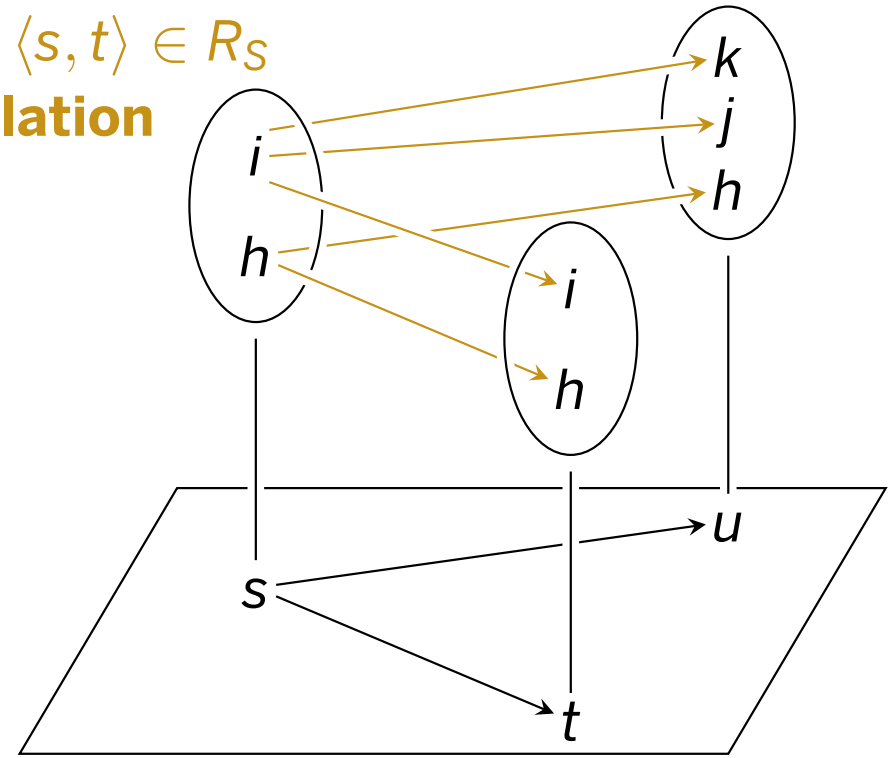

Path-lifting modal predicate logic: Truth

Define formulas, terms, models as usual.

A **valuation** v **at a state** s
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(Use to define **truth** $F, I, s, v \Vdash \phi$ when ϕ is atomic.)

If t is accessible from s, then we extend individual accessibility ${}_sR_t$ to **valuation accessibility:**

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Hallmark consequences

$$(x = y) \not\models \Box(x = y)$$
$$(x = y) \wedge \Box\phi(x, y) \models \Box\phi(y, x)$$

Frame correspondence

Reflexivity (T)

$$\forall s.\ (s\,R\,s \wedge \forall x.\ x\,{}_sR_s\,x)$$

iff

$$\forall I, s, v, \phi.\ F, I, s, v \Vdash \Box\phi \rightarrow \phi$$

Transitivity (4)

$$\forall s, t, u.\ (s\,R\,t \wedge t\,R\,u) \rightarrow (s\,R\,u \wedge \forall x, y, z.\ (x\,{}_sR_t\,y \wedge y\,{}_tR_u\,z) \rightarrow x\,{}_sR_u\,z)$$

iff

$$\forall I, s, v, \phi.\ F, I, s, v \Vdash \Box\phi \rightarrow \Box\Box\phi$$

Conclusion

Not just splitting but also merging, growing, and tearing out hairs

A combinatorial explosion of discourse possibilities

> *"The single biggest problem in communication is the illusion that it has taken place." —George Bernard Shaw*

> *"The challenge with labels is when people stop thinking of them as conversation openers and think of them as conversation closers." —Lee Harrington*

What is model-theoretic reality?

A world is a path among states?

A thing is a path among individuals?