Embedded probabilistic programming

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Probabilistic inference

Model (what) | Inference (how)

$$\left.\begin{array}{l}\Pr(\text{Reality})\\ \Pr(\text{Obs} \mid \text{Reality})\\ \text{obs}\end{array}\right\} \Pr(\text{Reality} \mid \text{Obs} = \text{obs})$$

$$\Vert$$

$$\frac{\Pr(\text{Obs} = \text{obs} \mid \text{Reality}) \Pr(\text{Reality})}{\Pr(\text{Obs} = \text{obs})}$$

Declarative probabilistic inference

Model (what)	Inference (how)
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$$\left.\begin{array}{l}\Pr(\text{Reality})\\ \Pr(\text{Obs} \mid \text{Reality})\\ \text{obs}\end{array}\right\} \Pr(\text{Reality} \mid \text{Obs} = \text{obs}) \;\Big\| \; \frac{\Pr(\text{Obs} = \text{obs} \mid \text{Reality}) \Pr(\text{Reality})}{\Pr(\text{Obs} = \text{obs})}$$

Declarative probabilistic inference

	Model (what)	Inference (how)
Toolkit (BNT, PFP)	invoke →	distributions, conditionalization, . . .
Language (BLOG, IBAL, Church)	random choice, observation, . . .	← interpret

Declarative probabilistic inference

	Model (what)	Inference (how)
Toolkit (BNT, PFP)	+ use existing libraries, types, debugger	+ easy to add custom inference
Language (BLOG, IBAL, Church)	+ random variables are ordinary variables	+ compile models for faster inference

Declarative probabilistic inference

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Today: Best of both	invoke →	← interpret

Express models and inference as interacting programs in the same general-purpose language.

Declarative probabilistic inference

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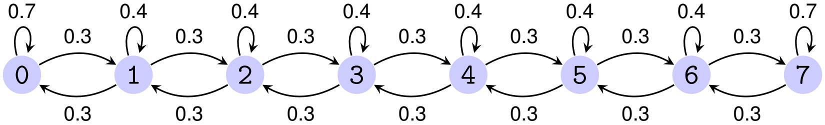
Toolkit (BNT, PFP)	+ use existing libraries, types, debugger	+ easy to add custom inference
Language (BLOG, IBAL, Church)	+ random variables are ordinary variables	+ compile models for faster inference
Today: Best of both	Payoff: expressive model + models *of inference*: bounded-rational theory of mind	Payoff: fast inference + deterministic parts of models run *at full speed* + importance sampling

Express models and inference as interacting programs in the same general-purpose language.

Outline

- **Expressivity**
  - Nested inference
- Implementation
  - Reifying a model into a search tree
  - Importance sampling with look-ahead
- Performance

Hidden Markov model



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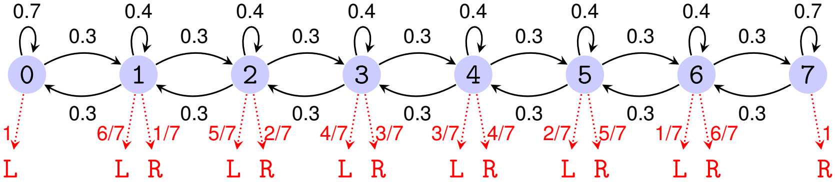

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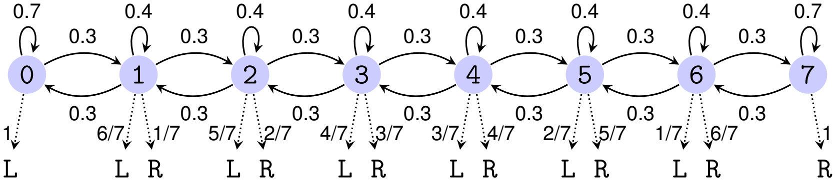
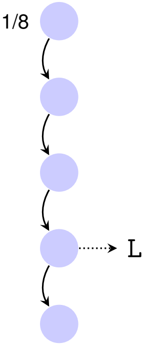
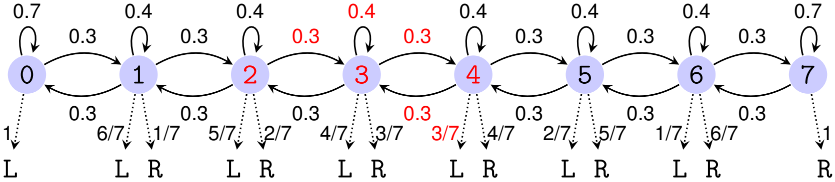
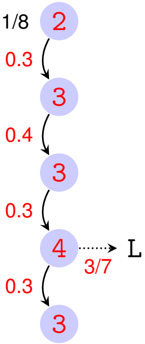




$$\Pr(\text{State}_5 \mid \text{Obs}_4 = \text{L})$$

Hidden Markov model





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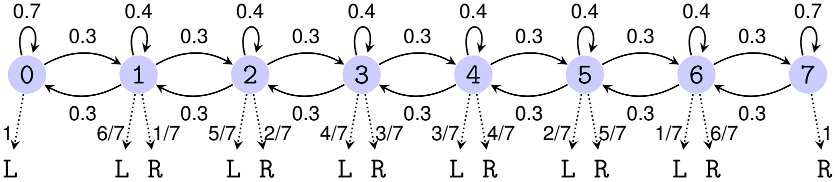
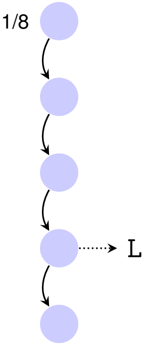
Hidden Markov model

`type state = int` `type obs = L | R` `let nstates = 8`





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Hidden Markov model

```
type state = int   type obs = L | R   let nstates = 8
let transition_prob = [| [(0.7,0); (0.3,1)]; ... |]
let evolve : state -> state = fun st ->
  dist (transition_prob.(st))
let observe : state -> obs = fun st ->
  let p = float st /. float (nstates - 1) in
  dist [(1.-.p, L); (p, R)]
let rec run = fun n obs ->
  let st = if n = 1 then uniform nstates else
           evolve (run (n - 1) obs) in
  obs st n; st
run 5
  (fun st n -> if n = 4 && observe st <> L then fail ())
```

Models are ordinary code (in OCaml) using a library function `dist`.

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Models are ordinary code (in OCaml) using a library function `dist`.
Inference applies to a thunk and returns a distribution.

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Models are ordinary code (in OCaml) using a library function `dist`.
Inference applies to a thunk and returns a distribution.
Deterministic parts of models run at full speed.

Models as programs in a general-purpose language

Reuse existing infrastructure!

- Rich libraries: lists, arrays, database access, I/O, . . .
- Type inference
- Functions as first-class values
- Compiler
- Debugger
- Memoization

  Express *Dirichlet processes*, etc. (Goodman et al. 2008)

  Speed up inference using lazy evaluation

  bucket elimination

  sampling w/memoization (Pfeffer 2007)

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Self application: nested *inference*

Choose a coin that is either fair or completely biased for `true`.

```
let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
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What is the probability that p is at least 0.3?

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Answer: 1.

```
at_least 0.3 true (exact_reify coin)
```

Self application: nested *inference*

```
exact_reify (fun () ->
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let biased = flip 0.5 in
let coin = fun () -> flip 0.5 || biased in
```

Let p be the probability that flipping the coin yields `true`.
Estimate p by flipping the coin twice.
What is the probability that our estimate of p is at least 0.3?
Answer: 7/8.

```
at_least 0.3 true (sample 2 coin)                )
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Self application: nested *inference*

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exact_reify (fun () ->
```

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Let p be the probability that flipping the coin yields `true`.
Estimate p by flipping the coin twice.
What is the probability that our estimate of p is at least 0.3?
Answer: 7/8.

```
at_least 0.3 true (sample 2 coin)                )
```

Returns a distribution—not just nested `query` (Goodman et al. 2008).
Inference procedures are OCaml code using `dist`, like models.
Works with observation, recursion, memoization.
Bounded-rational theory of mind **without interpretive overhead**.

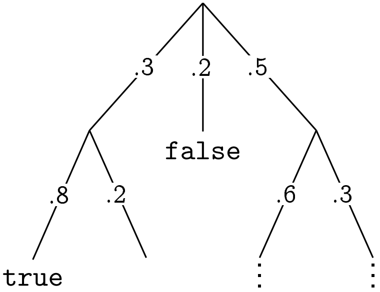
Outline

Expressivity
- Nested inference

▶ **Implementation**
- Reifying a model into a search tree
- Importance sampling with look-ahead

Performance

Reifying a model into a search tree



Exact inference by depth-first brute-force enumeration.
Rejection sampling by top-down random traversal.

Reifying a model into a search tree

open

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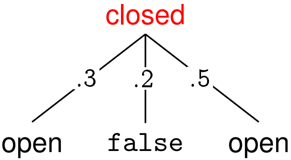
Reifying a model into a search tree



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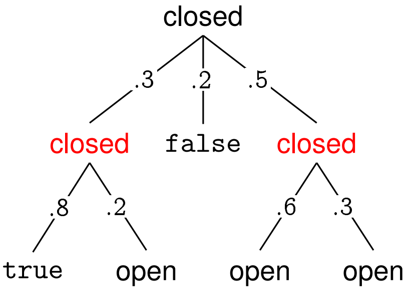

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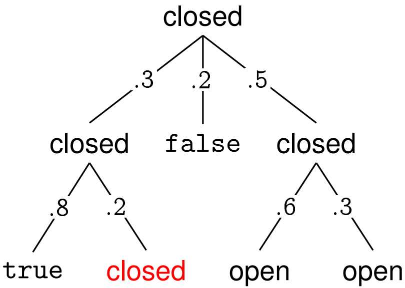

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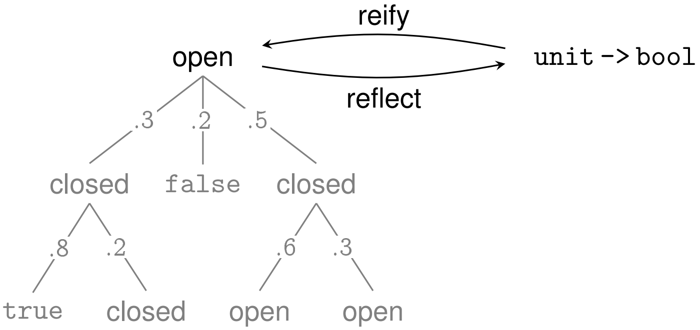
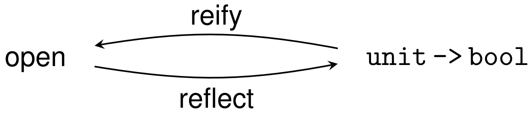


Inference procedures cannot access models' source code.

Reify then reflect (materialized views):

- Brute-force enumeration becomes bucket elimination
- Sampling becomes particle filtering

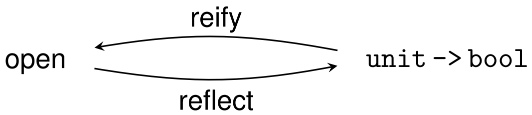
Reifying a model into a search tree



Implementation:

- represent a probability and state monad
  (Giry 1982, Moggi 1990, Filinski 1994)
- using first-class delimited continuations
  (Strachey & Wadsworth 1974,
  Felleisen et al. 1987,
  Danvy & Filinski 1989)

Reifying a model into a search tree



Implementation: shallow DSL embedding

```
let dist ch    = List.map ... ch
let literal x = unit x
let app e0 e1 = bind e0 (fun f -> bind e1 (fun x -> f x))
```

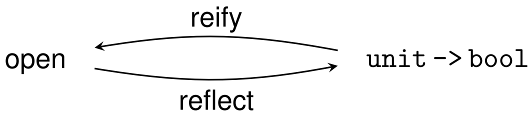
Continuation-passing style

```
let dist ch    = fun k -> List.map ...k... ch
let literal x = fun k -> k x
let app e0 e1 = fun k -> e0 (fun f -> e1 (fun x -> f x k))
```

First-class delimited continuations

```
let dist ch    = shift (fun k -> List.map ...k... ch)
let literal x = x
let app e0 e1 = e0 e1
```

Reifying a model into a search tree



Implementation: using clonable user-level threads

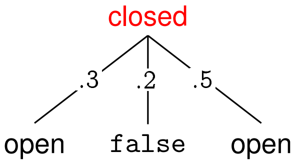
- Model runs inside a thread.
- `dist` clones the thread.
- `fail` kills the thread.
- Memoization mutates thread-local storage.

Analogy: Virtualize (not emulate) a CPU. Nesting works.

Importance sampling with look-ahead

open Probability mass $p_c = 1$

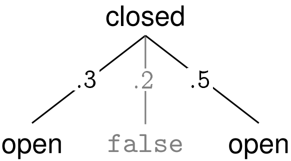
Importance sampling with look-ahead



Probability mass $p_c = 1$

1. Expand one level.

Importance sampling with look-ahead

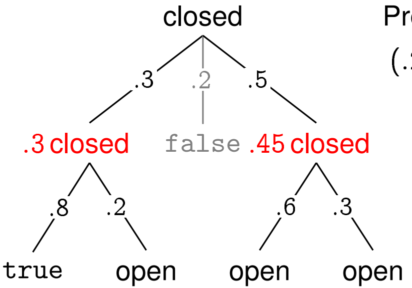


Probability mass $p_c = 1$

$(.2, \texttt{false})$

1. Expand one level.
2. Report shallow successes.

Importance sampling with look-ahead

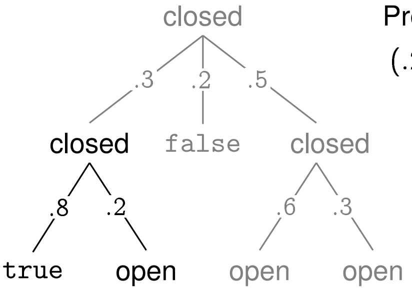


Probability mass $p_c = .75$

$(.2, \texttt{false})$

1. Expand one level.
2. Report shallow successes.
3. Expand one more level and tally open probability.

Importance sampling with look-ahead



Probability mass $p_c = .75$

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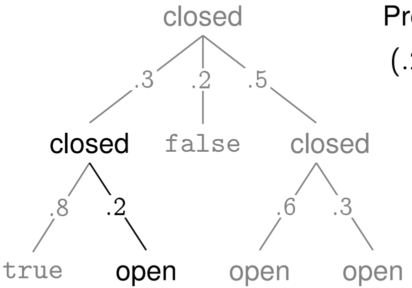
4. Randomly choose a branch and go back to 2.

Importance sampling with look-ahead



Probability mass $p_c = .75$

$(.2, \texttt{false})$ $(.6, \texttt{true})$

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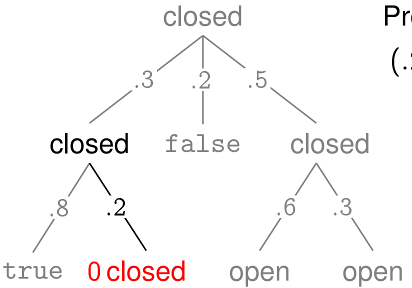
Importance sampling with look-ahead



Probability mass $p_c = 0$

$(.2, \texttt{false})$ $(.6, \texttt{true})$

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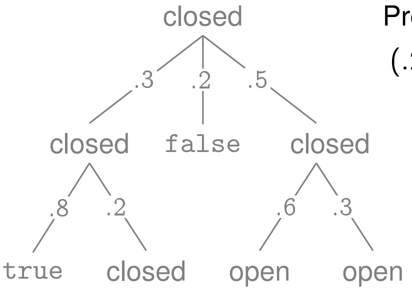

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- **Performance**

Motivic development in Beethoven sonatas

(Pfeffer 2007)

Source motif 

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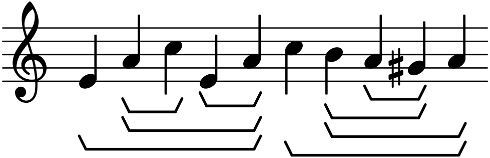

Motivic development in Beethoven sonatas

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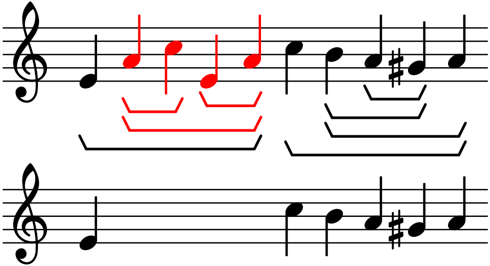

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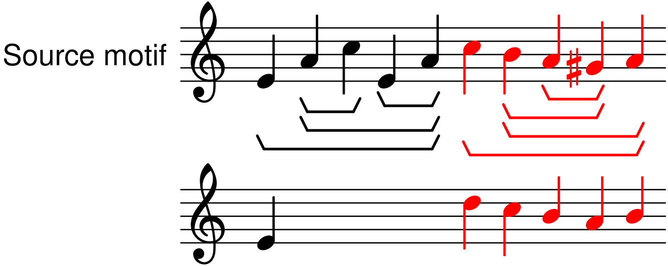

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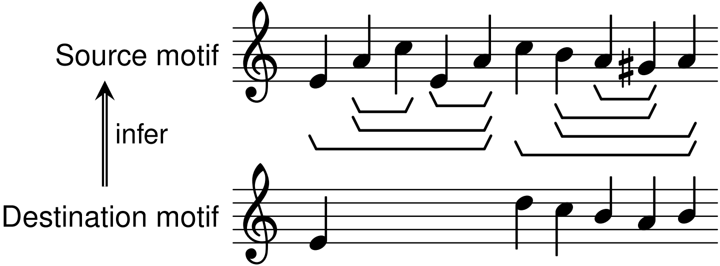

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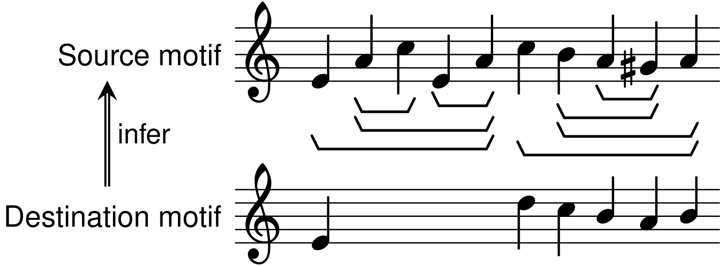

Implemented using lazy stochastic lists.

Motif pair	1	2	3	4	5	6	7
% correct using importance sampling							
• Pfeffer 2007 (30 sec)	93	100	28	80	98	100	63
• This paper (90 sec)	98	100	29	87	94	100	77
• This paper (30 sec)	92	99	25	46	72	95	61

Motivic development in Beethoven sonatas

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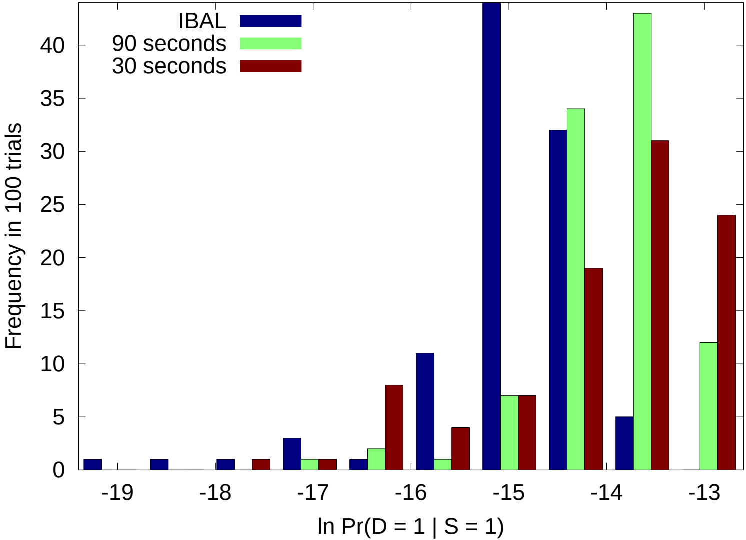
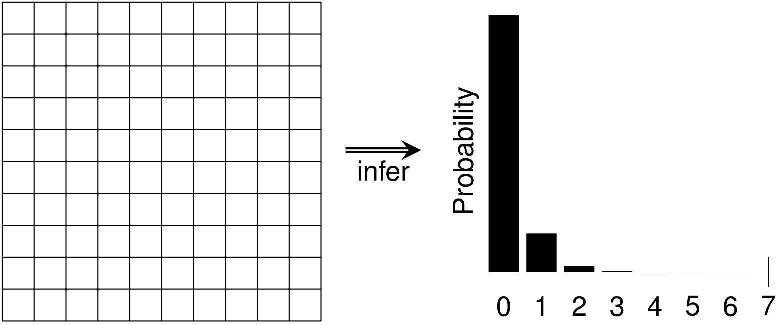


Noisy radar blips for aircraft tracking

(Milch et al. 2007)



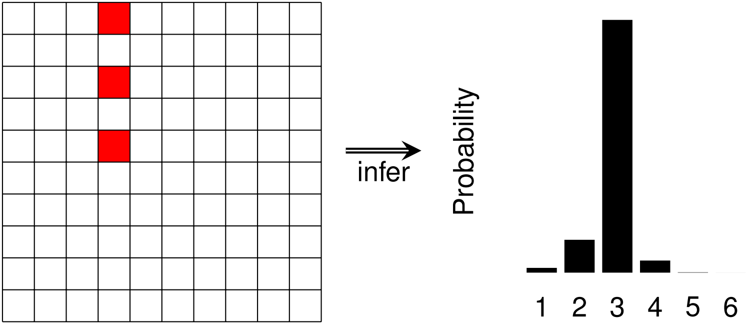
Blips present and absent

Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

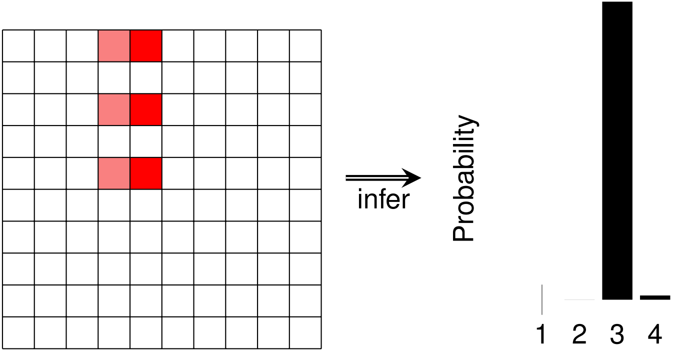
Number of planes

$t = 1$

Particle filter. Implemented using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

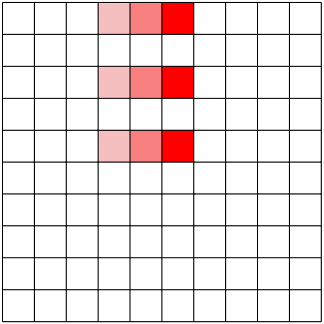
$t = 1, \quad t = 2$

Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

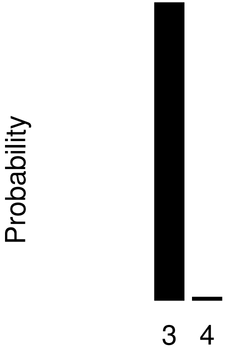
(Milch et al. 2007)



Blips present and absent

$t = 1, \quad t = 2, \quad t = 3$

$\Longrightarrow$ infer



Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Declarative probabilistic inference

	Model (what)	Inference (how)
Toolkit (BNT, PFP)	+ use existing libraries, types, debugger	+ easy to add custom inference
Language (BLOG, IBAL, Church)	+ random variables are ordinary variables	+ compile models for faster inference
Today: Best of both	Payoff: expressive model + models *of inference*: bounded-rational theory of mind	Payoff: fast inference + deterministic parts of models run *at full speed* + importance sampling

Express models and inference as interacting programs in the same general-purpose language.