Self-applicable probabilistic inference without interpretive overhead

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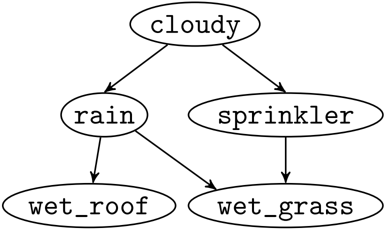
Probabilistic inference

Model (what) | Inference (how)

$$\left.\begin{array}{l}\Pr(\text{Reality})\\ \Pr(\text{Obs} \mid \text{Reality})\\ \text{obs}\end{array}\right\} \Pr(\text{Reality} \mid \text{Obs} = \text{obs})$$

$$\Vert$$

$$\frac{\Pr(\text{Obs} = \text{obs} \mid \text{Reality}) \Pr(\text{Reality})}{\Pr(\text{Obs} = \text{obs})}$$



$$\Pr(\texttt{rain} \mid \texttt{wet\_grass} = \texttt{true})$$

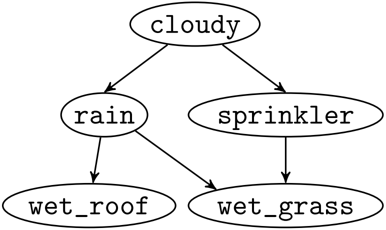
Declarative probabilistic inference

Model (what)

$\Pr(\text{Reality})$
$\Pr(\text{Obs} \mid \text{Reality})$
obs

Inference (how)

$$\Pr(\text{Reality} \mid \text{Obs} = \text{obs}) = \frac{\Pr(\text{Obs} = \text{obs} \mid \text{Reality}) \Pr(\text{Reality})}{\Pr(\text{Obs} = \text{obs})}$$



$\Pr(\texttt{rain} \mid \texttt{wet\_grass} = \texttt{true})$

Declarative probabilistic inference

	Model (what)	Inference (how)
Toolkit (BNT, PFP)	invoke →	distributions, conditionalization, ...
Language (BLOG, IBAL, Church)	random choice, observation, ...	← interpret

Declarative probabilistic inference

	Model (what)	Inference (how)
Toolkit (BNT, PFP)	+ use existing libraries, types, debugger	+ easy to add custom inference
Language (BLOG, IBAL, Church)	+ random variables are ordinary variables	+ compile models for faster inference

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Today: Best of both	invoke →	← interpret

Express models and inference as interacting programs in the same general-purpose language.

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Toolkit (BNT, PFP)	+ use existing libraries, types, debugger	+ easy to add custom inference
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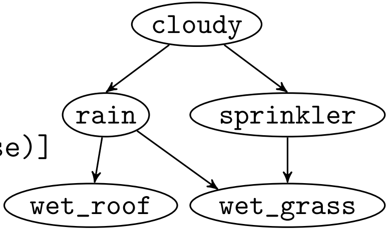
Today: Best of both	Payoff: expressive model + models *of inference*: bounded-rational theory of mind	Payoff: fast inference + deterministic parts of models run *at full speed* + importance sampling

Express models and inference as interacting programs in the same general-purpose language.

Outline

- **Expressivity**
  - Memoization
  - Nested inference
- Implementation
  - Reifying a model into a search tree
  - Importance sampling with look-ahead
- Applications

Grass model




```
let flip = fun p ->
 dist [(p, true); (1.-.p, false)]
```

Models are ordinary code (in OCaml) using a library function `dist`.

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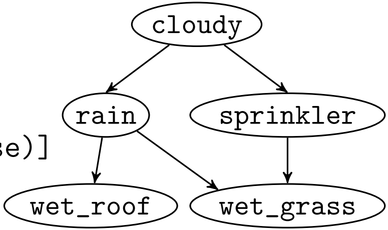


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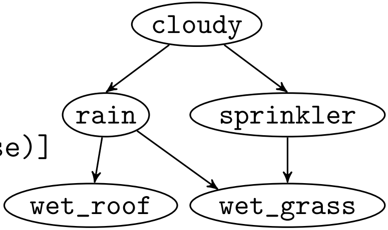


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 let cloudy    = flip 0.5 in
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 let wet_roof  = flip 0.7 && rain in
 let wet_grass = flip 0.9 && rain ||
                 flip 0.9 && sprinkler in
 if wet_grass then rain else fail ()
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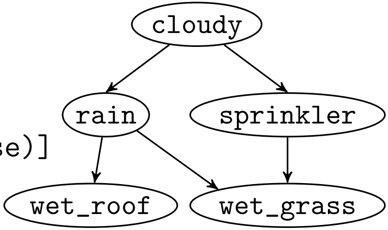


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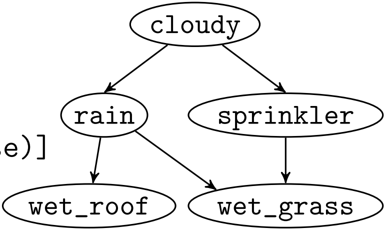


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normalize (exact_reify grass_model)
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Models are ordinary code (in OCaml) using a library function `dist`.
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Inference applies to *thunks* and returns a distribution.

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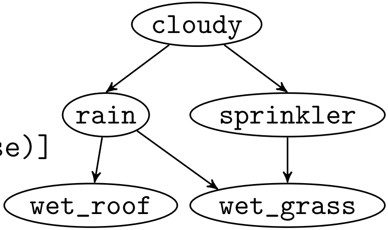


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Models are ordinary code (in OCaml) using a library function `dist`.
Random variables are ordinary variables.
Inference applies to *thunks* and returns a distribution.
Deterministic parts of models run at full speed.

Models as programs in a general-purpose language

Reuse existing infrastructure!

- Rich libraries: lists, arrays, database access, I/O, ...
- Type inference
- Functions as first-class values
- Compiler
- Debugger
- Memoization

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- Memoization

  Express Dirichlet processes, etc. (Goodman et al. 2008)

  Speed up inference using lazy evaluation
  bucket elimination
  sampling w/memoization (Pfeffer 2007)

Self application: nested *inference*

Choose a coin that is either fair or completely biased for `true`.

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let biased = flip 0.5 in
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Answer: 1.

```
at_least 0.3 true (exact_reify coin)
```

Self application: nested *inference*

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exact_reify (fun () ->
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Let p be the probability that flipping the coin yields `true`.
Estimate p by flipping the coin twice.
What is the probability that our estimate of p is at least 0.3?
Answer: 7/8.

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at_least 0.3 true (sample 2 coin)            )
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Self application: nested *inference*

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Answer: 7/8.

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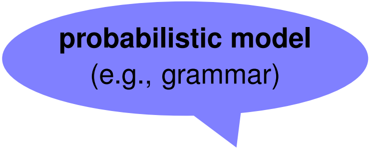
at_least 0.3 true (sample 2 coin)          )
```

Returns a distribution—not just nested `query` (Goodman et al. 2008).
Inference procedures are OCaml code using `dist`, like models.
Works with observation, recursion, memoization.
Bounded-rational theory of mind **without interpretive overhead**.

Grice and Marr



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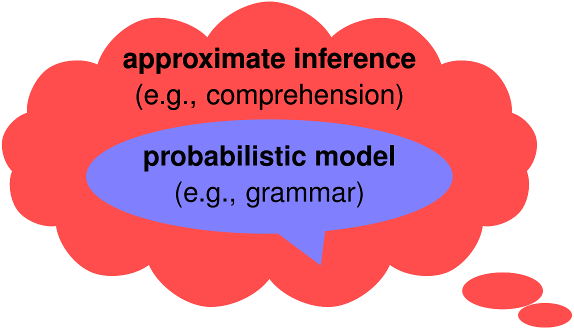

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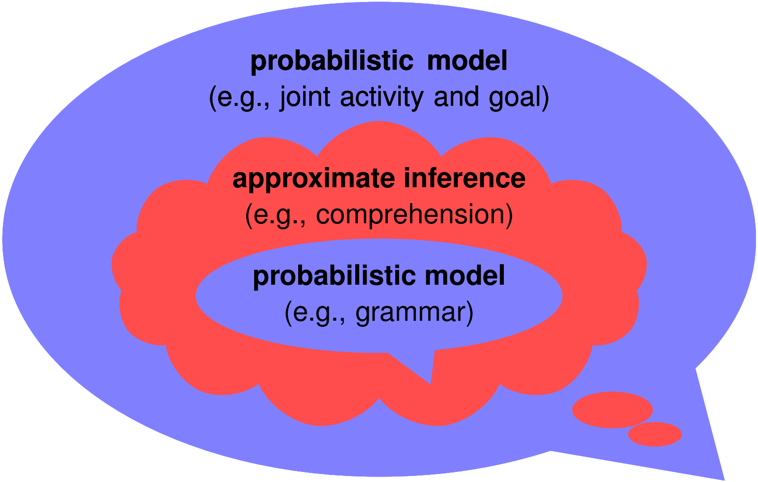

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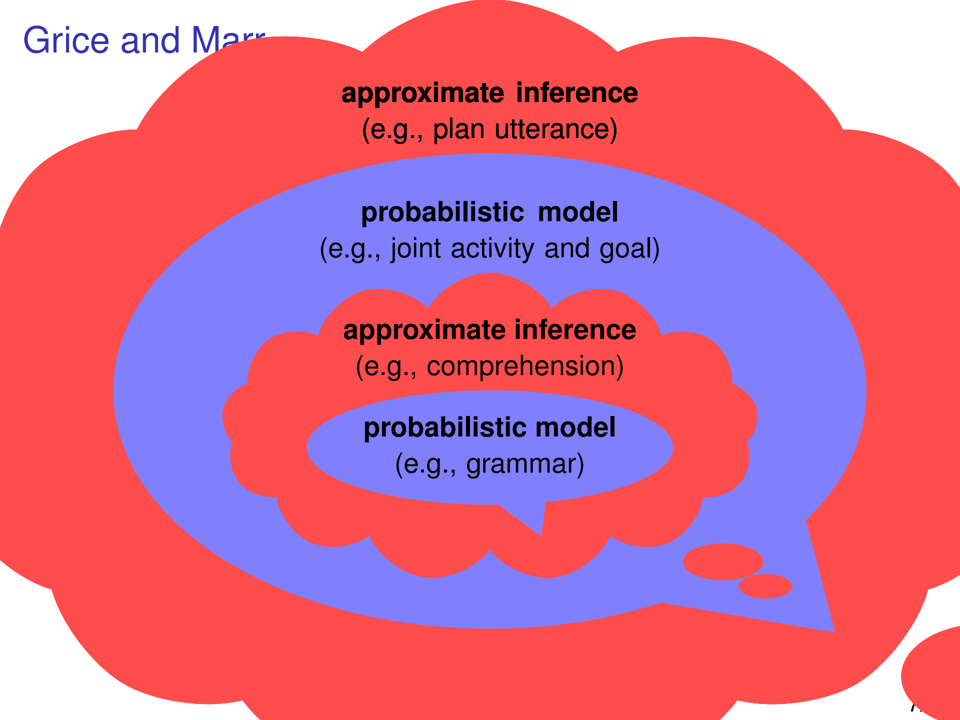

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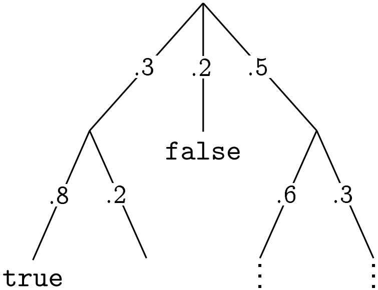
- **Implementation**
  - Reifying a model into a search tree
  - Importance sampling with look-ahead
- Applications

Reifying a model into a search tree



Exact inference by depth-first brute-force enumeration.
Rejection sampling by top-down random traversal.

Reifying a model into a search tree

open

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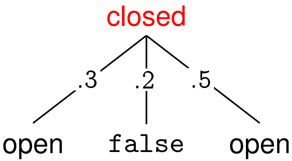

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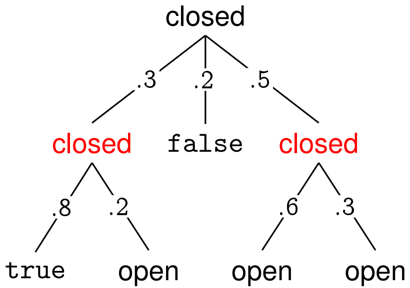

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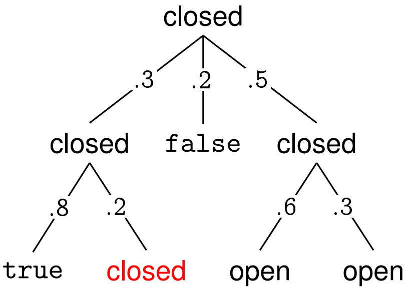

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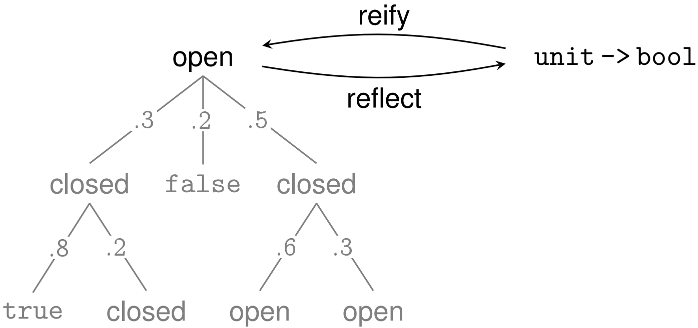
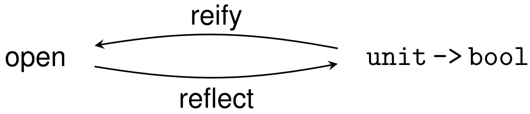


Inference procedures cannot access models' source code.

Reify then reflect:

- Brute-force enumeration becomes bucket elimination
- Sampling becomes particle filtering

Reifying a model into a search tree



Implementation: represent a probability and state monad
(Giry 1982, Moggi 1990, Filinski 1994)
using first-class delimited continuations
(Strachey & Wadsworth 1974,
Felleisen et al. 1987,
Danvy & Filinski 1989)

Implementation: using clonable user-level threads

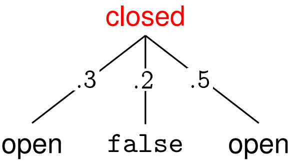
- Model runs inside a thread.
- `dist` clones the thread.
- `fail` kills the thread.
- Memoization mutates thread-local storage.

Analogy: Virtualize (not emulate) a CPU. Nesting works.

Importance sampling with look-ahead

open Probability mass $p_c = 1$

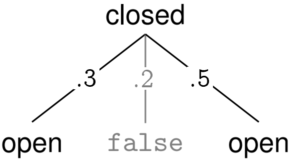
Importance sampling with look-ahead



Probability mass $p_c = 1$

1. Expand one level.

Importance sampling with look-ahead

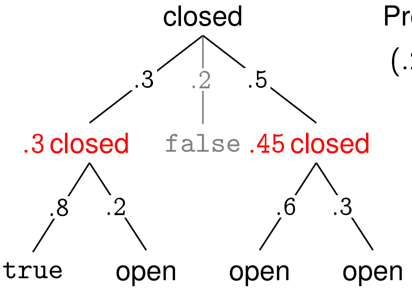


Probability mass $p_c = 1$

$(.2, \texttt{false})$

1. Expand one level.
2. Report shallow successes.

Importance sampling with look-ahead

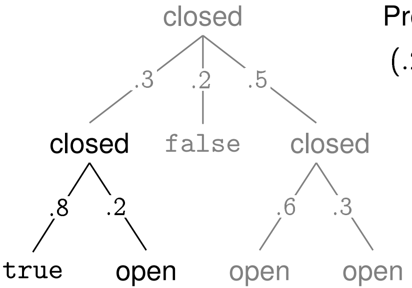


Probability mass $p_c = .75$

$(.2, \texttt{false})$

1. Expand one level.
2. Report shallow successes.
3. Expand one more level and tally open probability.

Importance sampling with look-ahead



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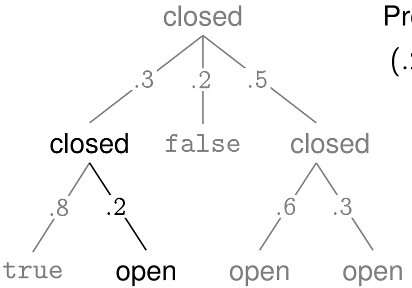

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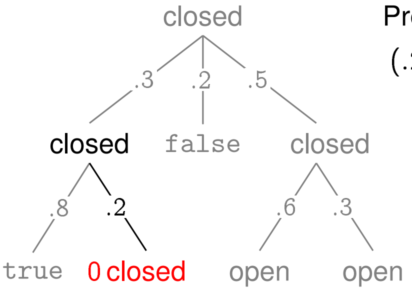

Probability mass $p_c = 0$

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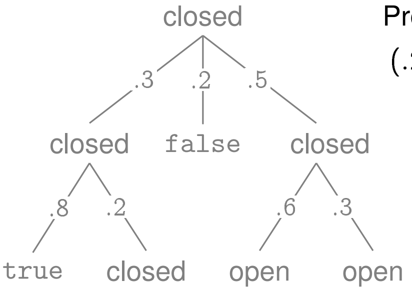

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- **Applications**

Conversational implicature in coordination discourse

Alice: some of our kids are coming home for dinner tonight.

Bob: (cooks food for $n-1$ kids)

Conversational implicature in coordination discourse

Linguist: Does ‘some’ mean ‘some but not all’?

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—trade off informativity against complexity

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Conversational implicature in coordination discourse

Linguist: Does ‘some’ mean ‘some but not all’?
—express nested probabilistic models intuitively

Alice: some of our kids are coming home for dinner tonight.
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The bounded-rational hearer's program

```
let count = (if flip 0.5 then 2 else 0) +
            (if flip 0.5 then 1 else 0) in
let conjunction = flip 0.5 in
if (not (some && not_all) || conjunction) &&
   (not some              || count > 0  ) &&
   (not not_all           || count < 3  )
then let action = ... in (action, utility action)
else fail ()
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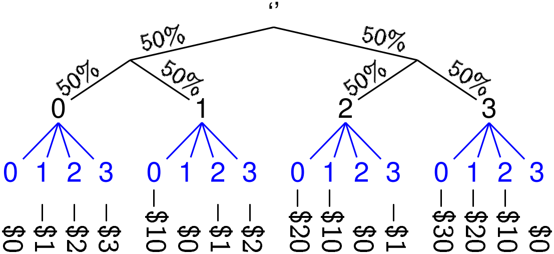

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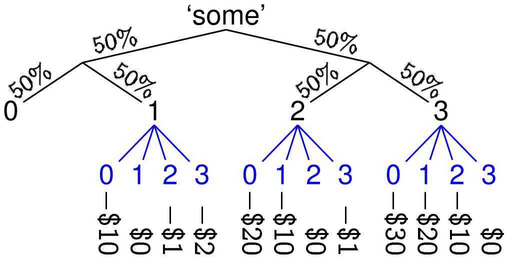

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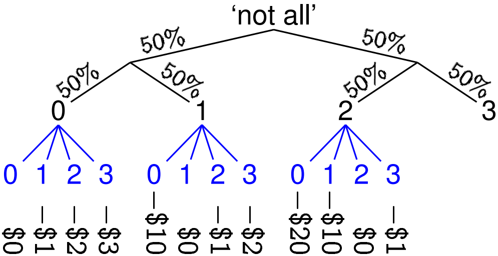

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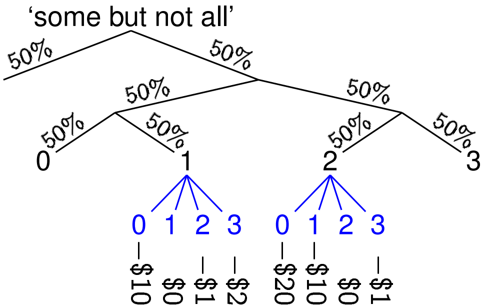

Motivic development in Beethoven sonatas

(Pfeffer 2007)

Source motif

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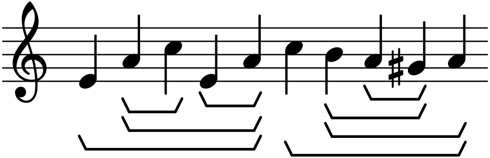

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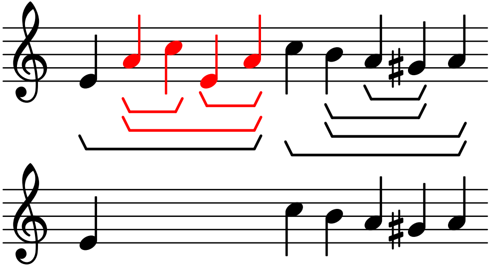

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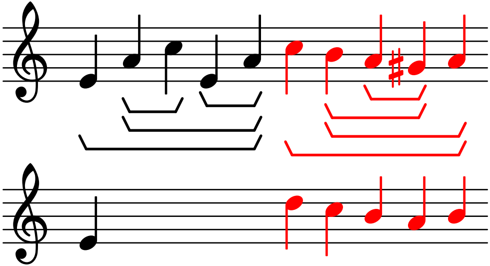
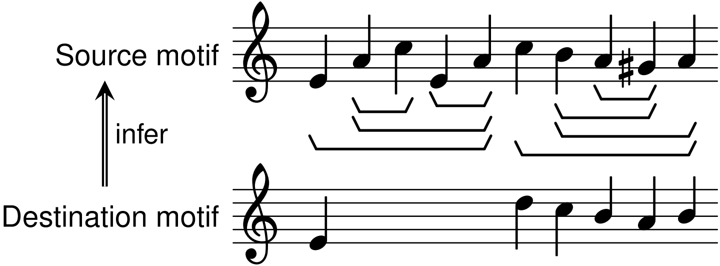
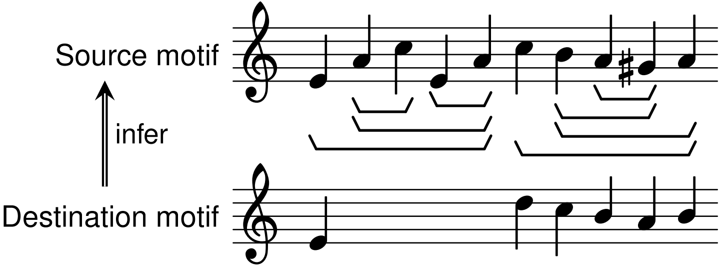


Motivic development in Beethoven sonatas

(Pfeffer 2007)





Implemented using lazy stochastic lists.

Motif pair		1	2	3	4	5	6	7
% correct using importance sampling								
• Pfeffer 2007	(30 sec)	93	100	28	80	98	100	63
• Us	(90 sec)	98	100	29	87	94	100	77
• Us	(30 sec)	92	99	25	46	72	95	61

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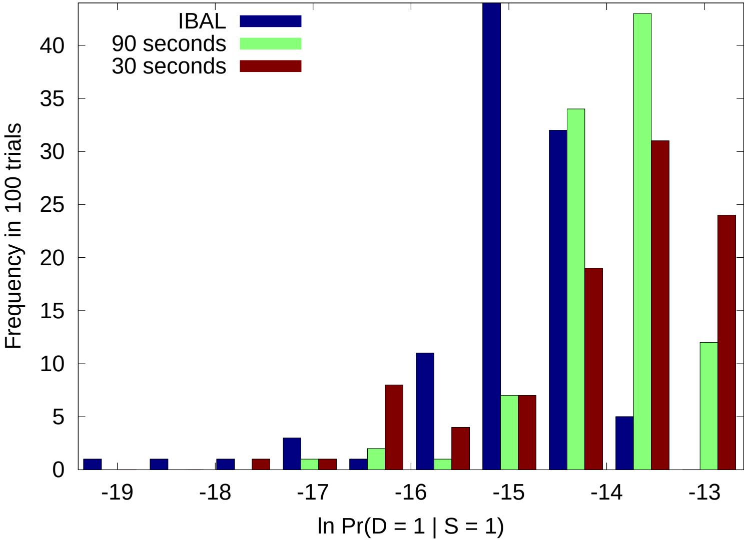
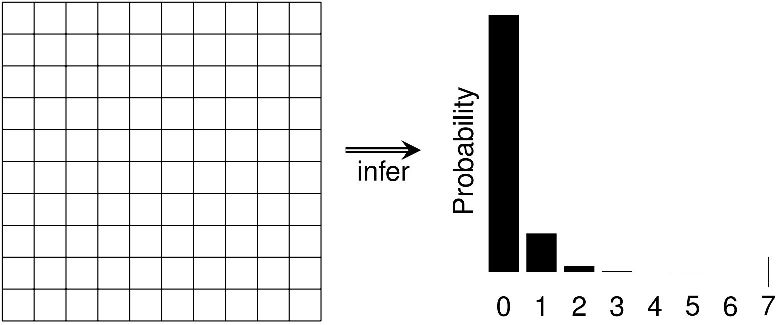


Noisy radar blips for aircraft tracking

(Milch et al. 2007)



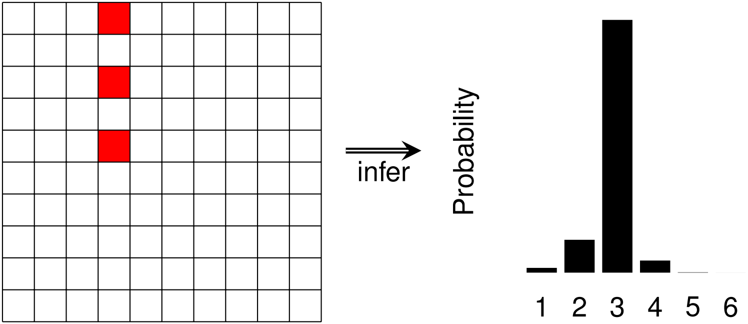
Blips present and absent

Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

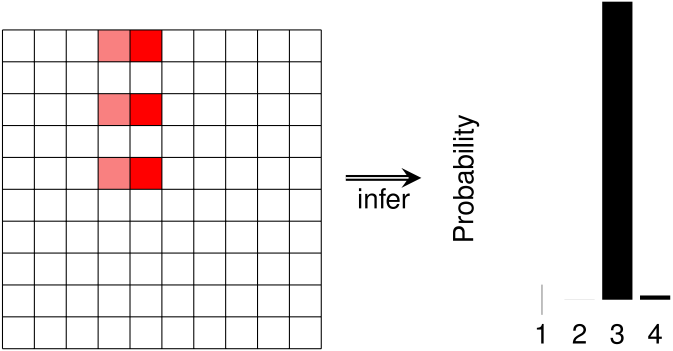
$t = 1$

Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

(Milch et al. 2007)



Blips present and absent

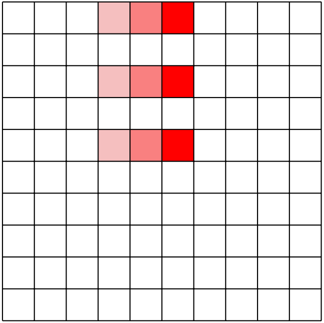
$t = 1, \quad t = 2$

Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Noisy radar blips for aircraft tracking

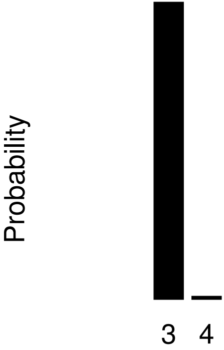
(Milch et al. 2007)



Blips present and absent

$t = 1, \quad t = 2, \quad t = 3$

infer



Number of planes

Particle filter. Implemented using lazy stochastic coordinates.

Summary

	Model (what)	Inference (how)
Toolkit	+ use existing libraries, types, debugger	+ easy to add custom inference
Language	+ random variables are ordinary variables	+ compile models for faster inference
Today: Best of both	Payoff: expressive model + models *of inference*: bounded-rational theory of mind	Payoff: fast inference + deterministic parts of models run *at full speed* + importance sampling

Express models and inference as interacting programs in the same general-purpose language.

HANSEI `http://okmij.org/ftp/kakuritu/`