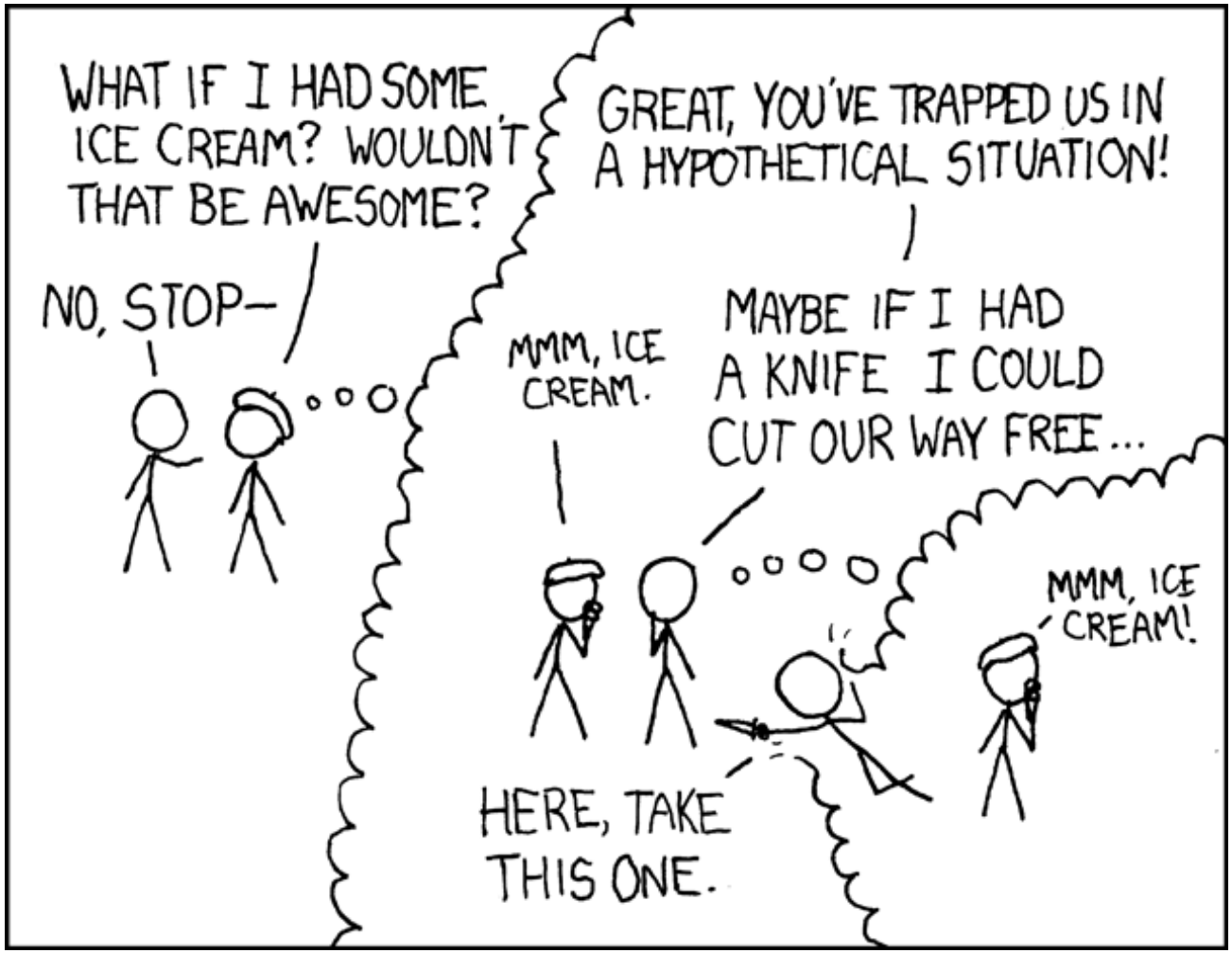
Theory of mind and bounded rationality without interpretive overhead

Chung-chieh Shan (Rutgers ⇄ Aarhus), with Oleg Kiselyov



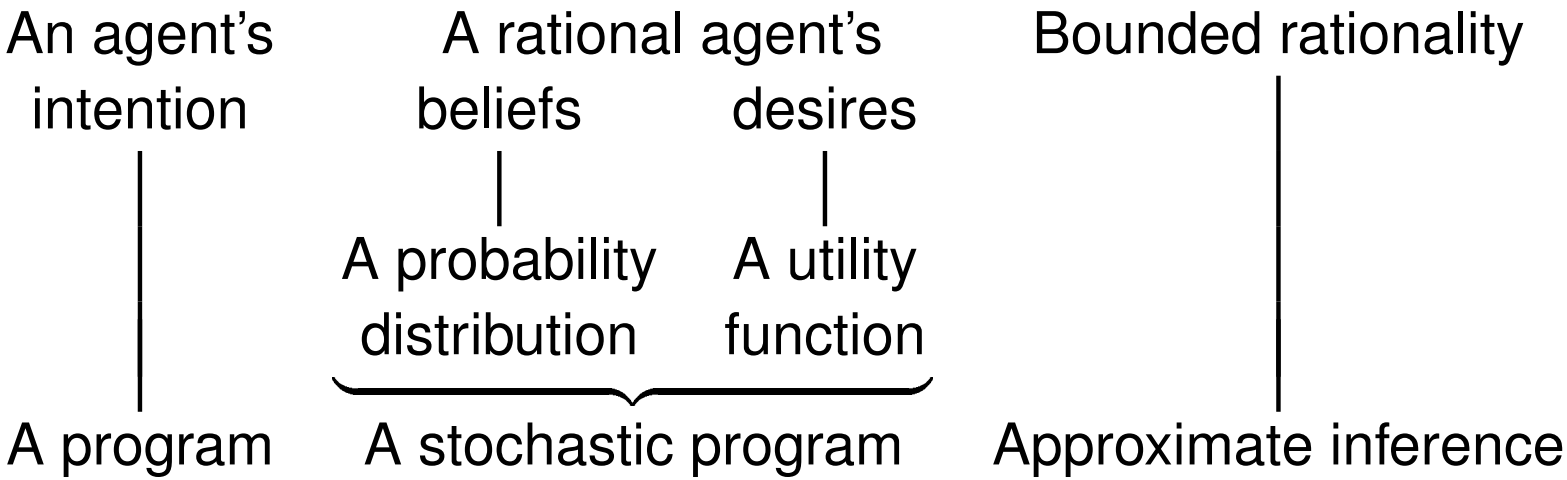
Theory of mind

- False-belief (Sally-Anne) task
- Gricean reasoning
- *p*-beauty contest
- Focal points in coordination games
- Information cascade
- Securities trading
- Plausibly deniable bribing (Pinker, Nowak, Lee)

Crucial for collaboration among human and computer agents!

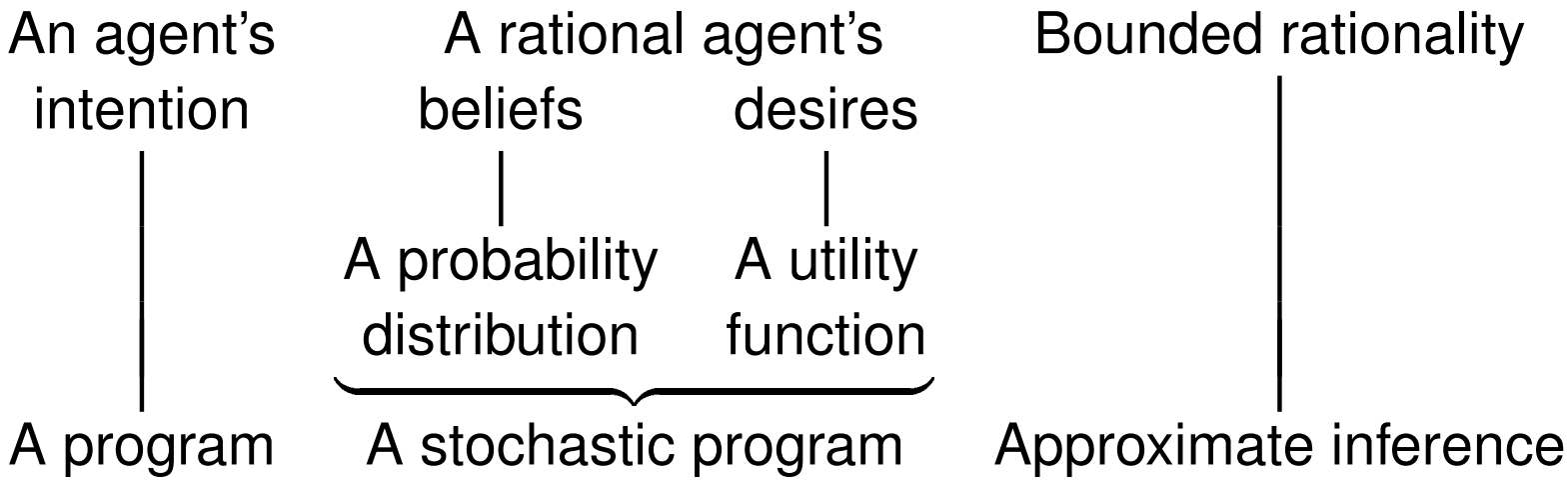
Want executable models.

Modeling minds as programs



The amount of detail varies.

Modeling minds as programs

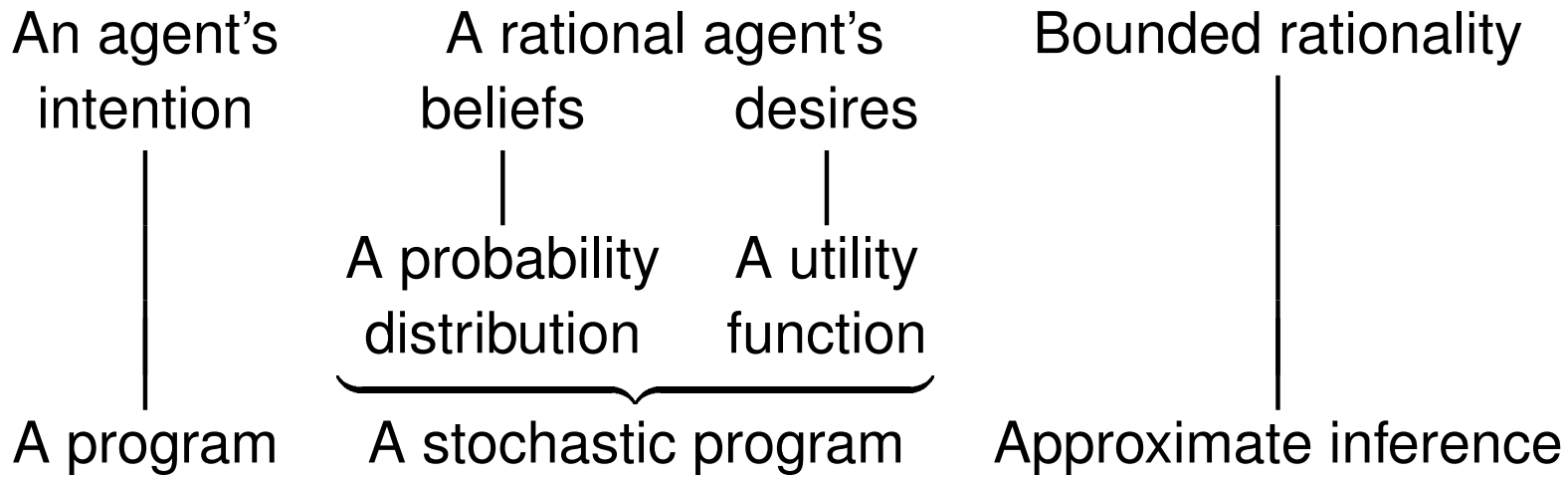


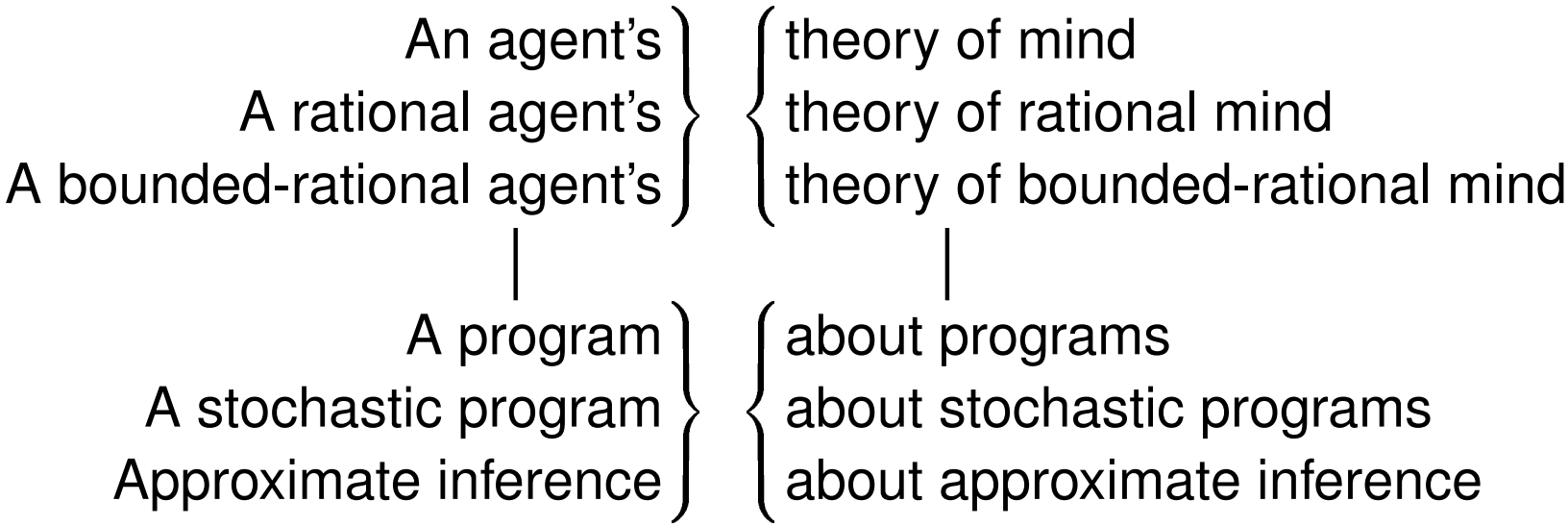

```
val random : random
val dist   : random -> (prob * 'a) list -> 'a
val fail   : random -> 'a

let flip random p = dist random [p,true; 1.-.p,false] in
let x = flip random 0.5 in
let y = flip random 0.5 in
if x || y then (x,y) else fail random
```

The amount of detail varies.

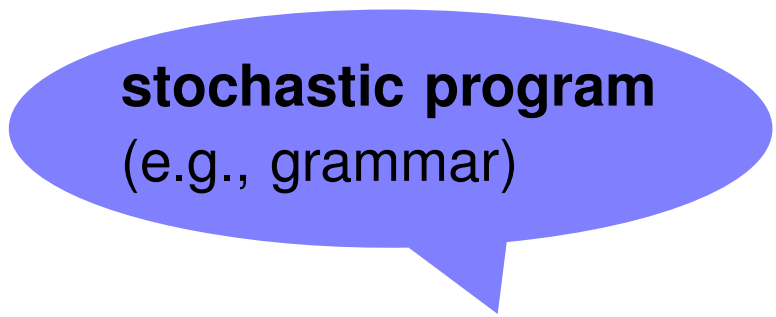
Modeling minds as programs





The amount of detail varies. Encapsulated weighted search.

Marr's computational vs algorithmic models



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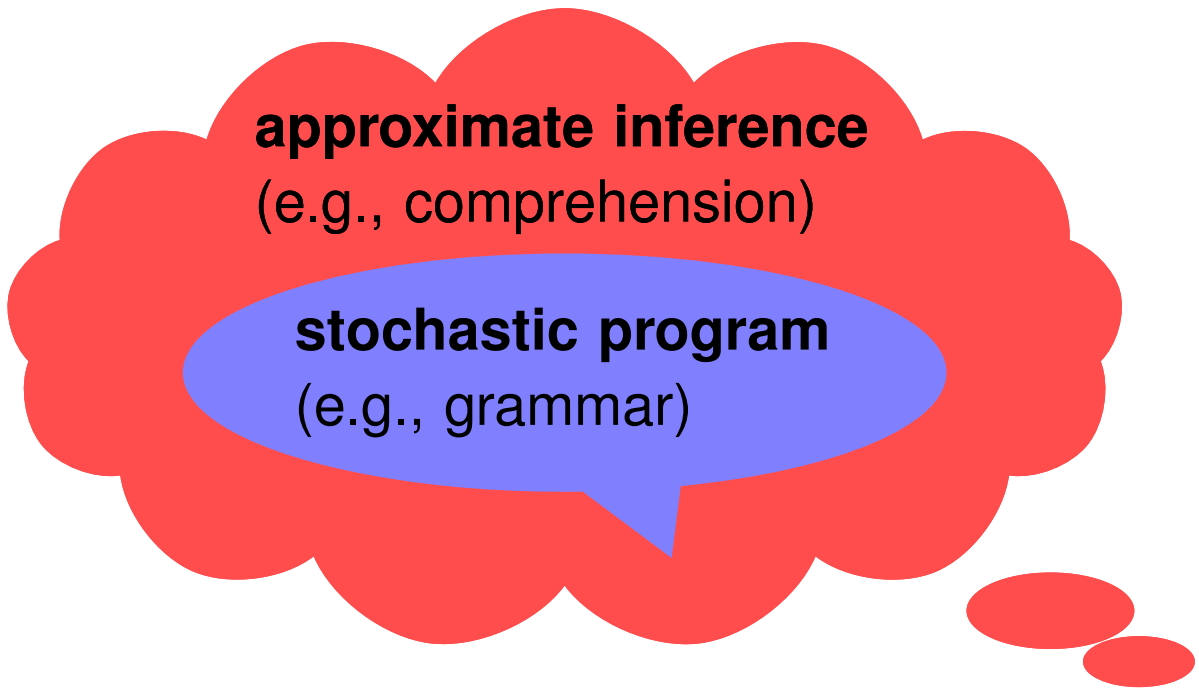

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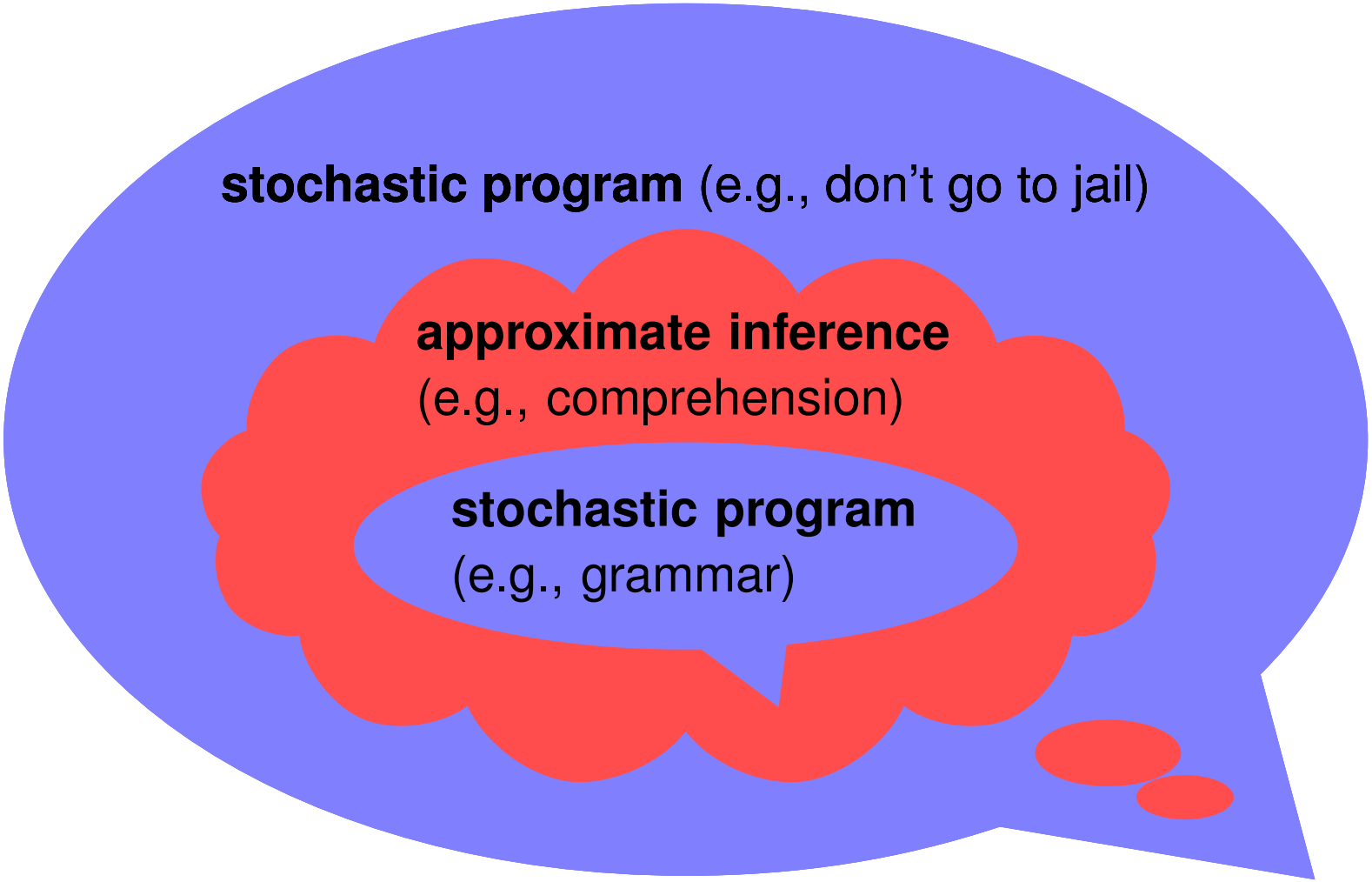
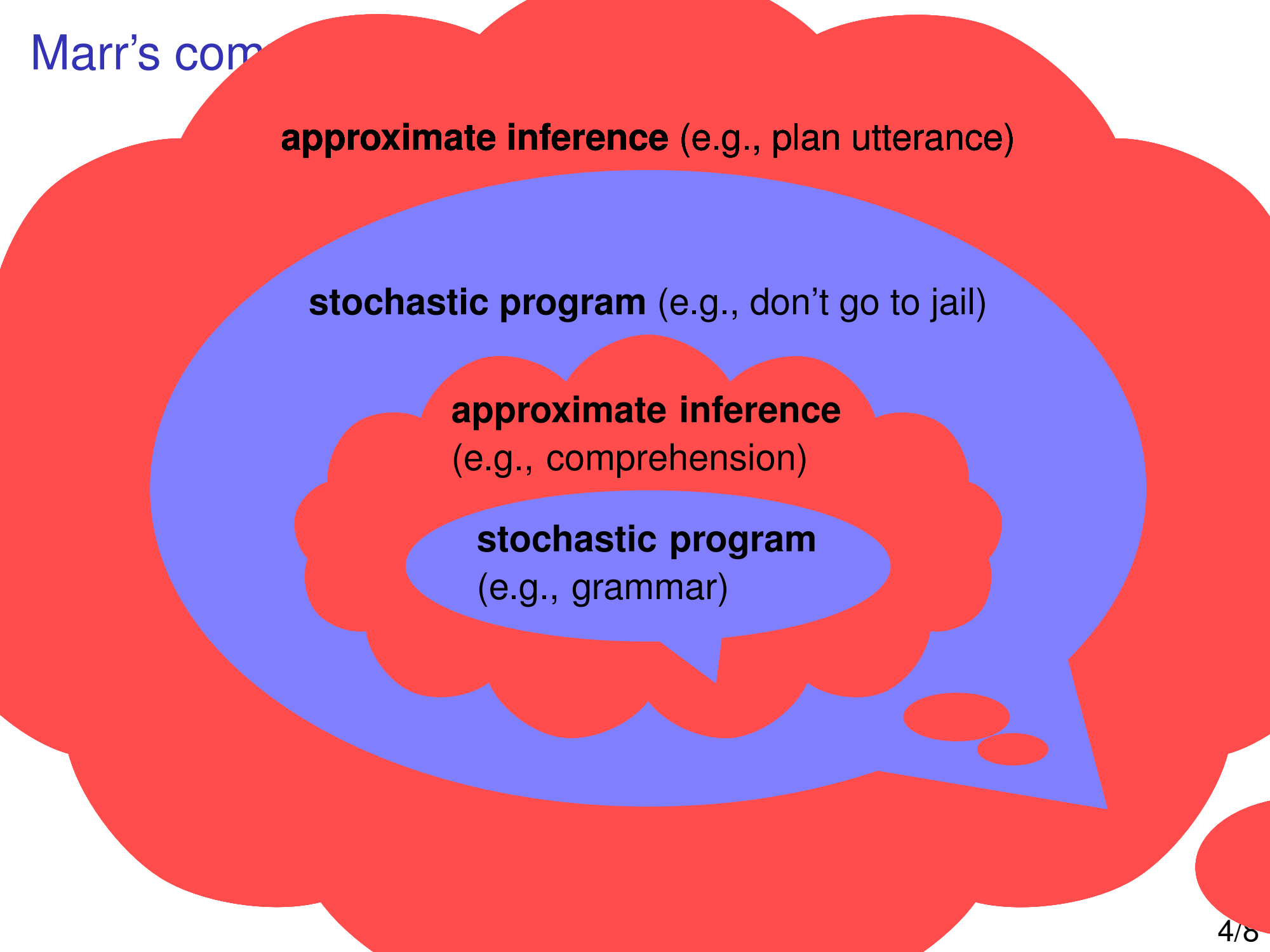
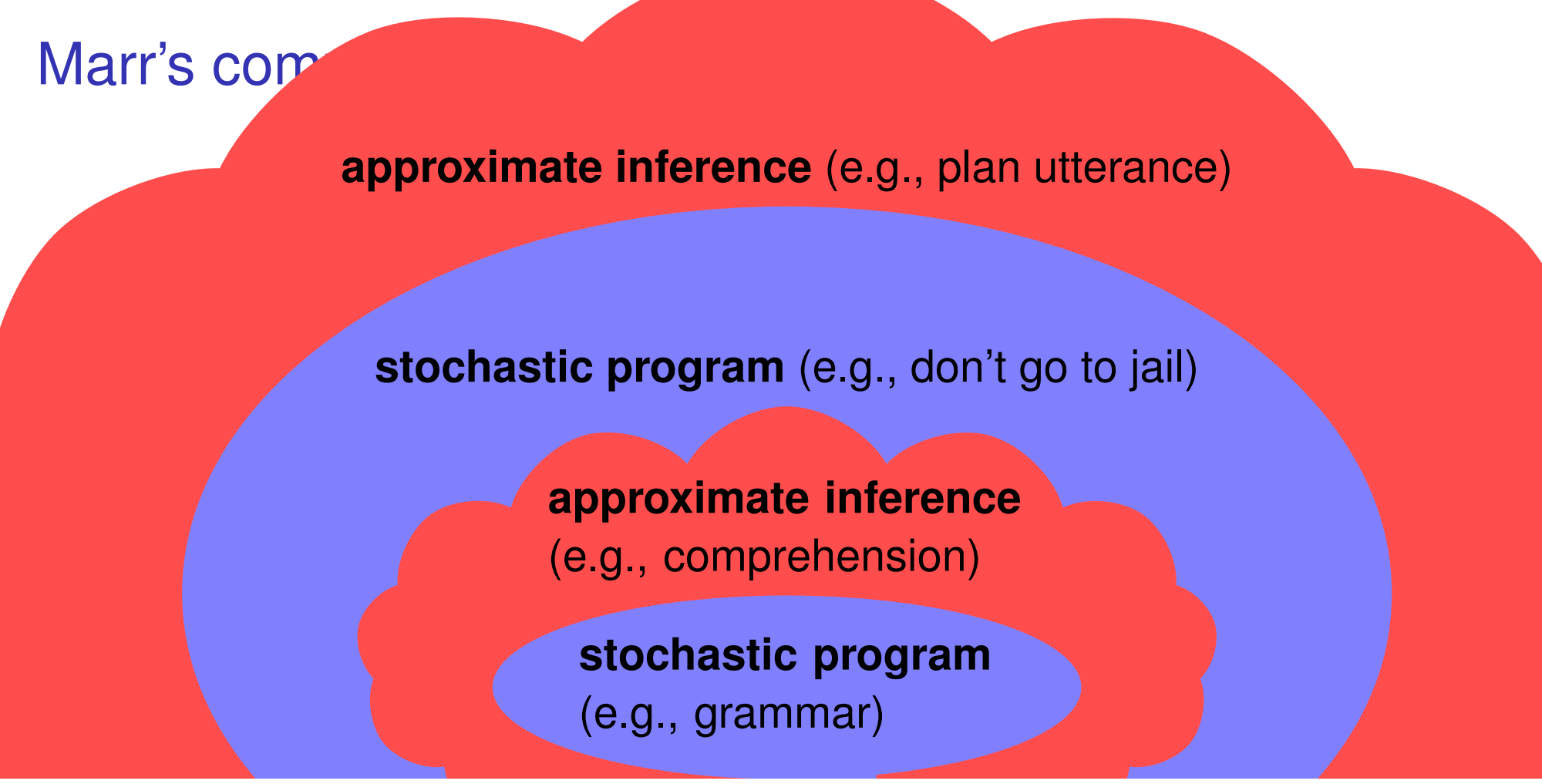


Marr's com


approximate inference (e.g., plan utterance)
stochastic program (e.g., don't go to jail)
approximate inference
(e.g., comprehension)
stochastic program
(e.g., grammar)

Marr's com



A computational model of the modeler nests an algorithmic model of the modelee.

For arbitrary nesting, implement inference as a stochastic program.

☞ Run deterministic code at full speed, to avoid slowdown exponential in the nesting depth (e.g., quantifier depth, plys).

How to eliminate interpretive overhead

Filinski: given a programming language with *delimited control*, add *layered side effects* (probabilities, memoization, etc.) while still running deterministic code at full speed.

- With delimited control, threads of execution can be suspended, resumed, copied, discarded.
- Represent stochastic programs not as data but as normal programs that suspend when they want randomness.
- Convert a stochastic program to a lazy tree of execution traces without interpretive overhead.
- Inference operates on this lazy tree. Implemented in OCaml.
- Inference is itself a stochastic program (e.g., importance sampling): it suspends when it wants randomness.
- Intuitions for nesting: sandboxes, virtualization, randomness adapters, mock objects.

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What comes in the box?

Represent stochastic programs

as normal programs

using `dist fail laze delay`

Represent approximate inference

as exploring a lazy tree of execution traces

using `sample_reify exact_reify collate at_least`

Represent theory of mind

as recursive invocations of approximate inference

using multiple randomness sources (values of type `random`)

Example: plausibly deniable bribing

```
# generate random ;;
.<fun random ->
  let make_boolean () =
    laze random (fun () -> flip random 0.5) in
  let q_1 = make_boolean () in
  let q_2 = make_boolean () in
  let q_3 = make_boolean () in
  let q_4 = make_boolean () in
  let q_5 = make_boolean () in
  if not (q_4 () <> q_3 () <> q_1 ()) &&
         (q_2 () <> q_5 () <> q_4 ()) &&
     not (q_2 () <> q_3 () <> q_2 ()) &&
     not (q_3 () <> q_3 () <> q_5 ()) &&
         (q_3 () <> q_4 () <> q_5 ())
  then q_5 () && q_4 ()
  else fail random>.
```

Example: plausibly deniable bribing

```
let predict random innocent problem =
  match collate
          (sample_reify random (Some 2) 5 problem)
  with
  | [] -> fail random                    (* police rejects sentence *)
  | [_, false] ->
    if innocent then Ticketed                  (* naïve driver *)
    else              (* police perceives (unambiguous) bribe *)
    if flip random 0.5 then Bribe          (* corrupt police *)
    else                                    (* honest police *)
    if at_least 0.01 true                   (* criminal trial *)
        (sample_reify random (Some 4) 20 problem)
    then Ticketed              (* court finds reasonable doubt *)
    else Convicted                          (* court finds bribe *)
  | _ -> Ticketed           (* police does not perceive bribe *)
```

Example: plausibly deniable bribing

```
let prefer random = function
  | Ticketed  -> if flip random 0.2 then fail random
  | Bribe     -> ()
  | Convicted -> fail random

let analyze problem =
  List.map snd (exact_reify problem)

let driver innocent random =
  let problem = .!(generate random) in
  prefer random (predict random innocent problem);
  analyze problem
```

Example: r

```
(* innocent driver *)
# collate (sample_reify random (Some 3) 1000
                        (driver true)) ;;
[(0.0724, [true; false]);                    (* 21% *)
 (0.0498, [true]);                           (* 15% *)
 (0.2183, [false])]                          (* 64% *)

(* bribing driver *)
# collate (sample_reify random (Some 3) 1000
                        (driver false)) ;;
[(0.0768, [true; false]);                    (* 30% *)
 (0.0457, [true]);                           (* 18% *)
 (0.1327, [false])]                          (* 52% *)
```

Summary

People people model model

- Concise, concrete, composable, compilable
- Can model unknown nesting depth

Next steps

- Applications please!
- Faster inference: conditional independence; memoization; Markov chain Monte Carlo
- From probability to expected value and maximum utility
- Imperfect information: staging