# How to reify fresh type variables?

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# Waiter, there's a term in my type!

Rank-2 polymorphism (ab)used, for safety:

- mutable state `runST :: (∀s. ST s a) -> a`
- array index bounds
- environment classifiers
- staged lexical scope
- resource control
- automatic differentiation

$$\dfrac{\begin{array}{c}[\alpha : \star] \\ \vdots \\ E : \tau\end{array}}{\Lambda\alpha.\, E : \forall\alpha.\, \tau}\ \forall\mathrm{I}$$

$\alpha$ is fresh, but the program doesn't know it.
Type-level gensyms for expressivity?

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- staged lexical scope — term gensym $\to$ type gensym
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$\forall$, see you later.

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R. Clint Whaley, ATLAS documentation:

> *You may have a naturally strong and negative reaction to these crude mechanisms, tempting you to send messages decrying my lack of humanity, decency, and legal parentage. . .   The proper bitch format involves*
>
> First *thanking me for spending time in hell*
> *getting things to their present crude state*
> Then, *supplying your constructive ideas*

# Higher-order abstract syntax

- `lam (\x -> x)` $\leadsto$
  `Lam "x1" (Var "x1")`
- `lam (\x -> let body = x in lam (\x -> body))` $\leadsto$
  `Lam "x2" (let body = Var "x2" in lam (\x -> body))` $\leadsto$
  `Lam "x2" (lam (\x -> Var "x2"))` $\leadsto$
  `Lam "x2" (Lam "x3" (Var "x2"))`

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Effects (error, state, let-insertion, etc.) beyond binders are hard.

- `lam (\x -> throw "hello")` $\leadsto$ ???
- `lam (\x -> throw x)` $\leadsto$ ???

> *It seems rather difficult, if not impossible, to manipulate open code in a satisfactory manner when higher-order code representation is chosen.* *(Chen & Xi, JFP 2005)*

We need name generation, but dissociated from binding.

# Gensym

- `let x = gensym() in Lam x (Var x)` ⇝
  `Lam "x1" (Var "x1")`
- ```
  let x = gensym() in Lam x
   (let body = Var x in
    let x = gensym() in Lam x body)
  ```
  ⇝
  `Lam "x2" (Lam "x3" (Var "x2"))`
- `let x = gensym() in cogen (fun body -> Lam x body)` ⇝
  ```

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- `let x = gensym() in cogen (fun body -> Lam x body)` ⇝

Ruling out scope extrusion is hard.

- `let x = gensym() in Lam x (throw "hello")` ⇝
- `let x = gensym() in Lam x (throw (Var x))` ⇝
  ```

# So, de Bruijn

- `Lam Zero`
- `Lam (let body = Zero in Lam (Succ body))` $\rightsquigarrow$ `Lam (Lam (Succ Zero))`
- `let x = Zero in cogen (fun body -> Lam body)` $\rightsquigarrow$

# So, de Bruijn

- `Lam Zero`
- `Lam (let body = Zero in Lam (Succ body))` $\leadsto$
  `Lam (Lam (Succ Zero))`
- `let x = Zero in cogen (fun body -> Lam body)` $\leadsto$

Mourn the loss of HOAS beauty.

Meta-types should reflect object type judgments
(Nanevski, Pfenning & Pientka, TOCL 2008).

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\texttt{Zero} : (\Gamma, \texttt{Int} \vdash \texttt{Int})}}{\texttt{Succ Zero} : (\Gamma, \texttt{Int}, \texttt{Bool} \vdash \texttt{Int})}}{\texttt{Lam (Succ Zero)} : (\Gamma, \texttt{Int} \vdash \texttt{Bool} \to \texttt{Int})}}{\texttt{Lam (Lam (Succ Zero))} : (\Gamma \vdash \texttt{Int} \to \texttt{Bool} \to \texttt{Int})}$$

# Type safety

Open code and closed code have distinct types:

$$\frac{\texttt{catch (throw (Lam Zero))} : (\vdash \texttt{Int} \to \texttt{Int})}{\texttt{run (catch (throw (Lam Zero)))} : \texttt{Int} \to \texttt{Int}}$$

$$\texttt{catch (Lam (throw "hello"))} : \texttt{String}$$

$$\texttt{catch (Lam (throw Zero))} : (\Gamma, \texttt{Int} \vdash \texttt{Int})$$

$$\frac{\dfrac{\texttt{catch (Lam (throw Zero))} : (\texttt{Int} \vdash \texttt{Int})}{\texttt{Lam (catch (Lam (throw Zero)))} : (\vdash \texttt{Int} \to \texttt{Int})}}{\texttt{run (Lam (catch (Lam (throw Zero))))} : \texttt{Int} \to \texttt{Int}}$$

(Kim, Yi & Calcagno, POPL 2006, §6.4)

Where did lexical scope go?

# Unexpectedly bound variables

`uneasy f = Lam (Lam (f Zero))` (Chen & Xi, JFP 2005)

- `uneasy id` ⇝ `Lam (Lam Zero)`
- `uneasy Succ` ⇝ `Lam (Lam (Succ Zero))`
- `uneasy (fun body -> Lam (Succ body))` ⇝ `Lam (Lam (Lam (Succ Zero)))`

> *In light of these examples, we claim that, perhaps contrary to popular belief, well-scopedness of de Bruijn indices is not good enough: it does not guarantee that indices are correctly adjusted where needed.*
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> *(Pouillard & Pottier, ICFP 2010)*

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(McBride)

# Safety in numbers

- `let x = gensym() in Lam x (Zero x)` ⇝
  `Lam 1 (Zero 1)`
- ```
  let x = gensym() in Lam x
   (let body = Zero x in
    let x = gensym() in Lam x (Succ body))
  ```
  ⇝
  `Lam 2 (Lam 3 (Succ (Zero 2)))`
- `let x = gensym() in cogen (fun body -> Lam x body)` ⇝
  ```

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  ```
- `let x = gensym() in cogen (fun body -> Lam x body)` ⇝

Lexical scope = labels all match.

- ```
  let x = gensym() in Lam x
   (catch (let y = gensym() in Lam y
             (throw (Zero x)))) ⇝
  Lam 4 (Zero 4)
  ```

# Safety in numbers

- `let x = gensym() in Lam x (Zero x)` ⇝
  `Lam 1 (Zero 1)`
- ```
  let x = gensym() in Lam x
   (let body = Zero x in
    let x = gensym() in Lam x (Succ body)) ⇝
  Lam 2 (Lam 3 (Succ (Zero 2)))
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- `let x = gensym() in cogen (fun body -> Lam x body)` ⇝

Lexical scope = labels all match.

- ```
  let x = gensym() in Lam x
   (catch (let y = gensym() in Lam y
              (throw (Zero y)))) ⇝
  Lam 4 (Zero 5)
  ```

# Meta-scope expresses binding expectations

```
uneasy f = let x = gensym() in Lam x
             (let y = gensym() in Lam y
               (f (Zero y)))
```

- `uneasy id` ⇝ `Lam 6 (Lam 7 (Zero 7))`
- `uneasy Succ` ⇝ `Lam 6 (Lam 7 (Succ (Zero 7)))`
- `uneasy (fun body ->`
  `let z = gensym() in Lam z (Succ body))` ⇝
  `Lam 6 (Lam 7 (Lam 8 (Succ (Zero 7))))`

Checking easily made compositional (incremental).

# Static capabilities

```
lam :: Functor m =>
       (∀s.       ((H Code s α, Γ) -> Code α)
            -> m ((H Code s α, Γ) -> Code β)       )
       ->     m (             Γ  -> Code (α->β))
```

Here `m` is the effect
`s` is the static proxy for the gensym, attached using `H`
$\alpha$ is the domain of the generated function
$\beta$ is the range of the generated function
$\Gamma$ is the type environment of the generated function

Claim: if the generator is well-typed, then the generated code is well-labeled.

# Static capabilities

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Claim: if the generator is well-typed, then the generated code is well-labeled.

- How to reify type-level gensym?
- How to unify compile-time and run-time gensym?
- How to automate weakening?

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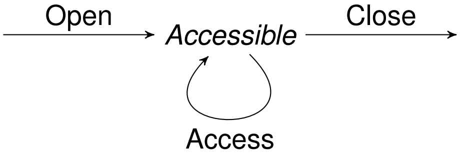
# Lightweight monadic regions (Haskell 2008)

## Goal: Resource management

- No access after close (down with run-time checking)
- Timely disposal (especially for scarce resources)
- Error handling



# Type-state

```
test h1 = do h2 <- hOpen "config" ReadMode
             fname <- hGetLine h2
             h3 <- hOpen fname WriteMode
             hPutStrLn h3 fname
             till (liftM2 (||) (hIsEOF h2) (hIsEOF h1))
                  (hGetLine h2 >>= hPutStrLn h3 >>
                   hGetLine h1 >>= hPutStrLn h3)
             hClose h2
             return h3
```

# Type-state

```
test :: Handle -> IO Handle

test h1 = do h2 <- hOpen "config" ReadMode
             fname <- hGetLine h2
             h3 <- hOpen fname WriteMode
             hPutStrLn h3 fname
             till (liftM2 (||) (hIsEOF h2) (hIsEOF h1))
                  (hGetLine h2 >>= hPutStrLn h3 >>
                   hGetLine h1 >>= hPutStrLn h3)
             hClose h2
             return h3
```

# Type-state

```
class Monadish m where
  return :: a -> m p p a
  (>>=)  :: m p q a -> (a -> m q r b) -> m p r b

test :: SHandle 0 ->
        SIO (1,[0]) (3,[2,0]) (SHandle 2)
test h1 = do h2 <- hOpen "config" ReadMode
             fname <- hGetLine h2
             h3 <- hOpen fname WriteMode
             hPutStrLn h3 fname
             till (liftM2 (||) (hIsEOF h2) (hIsEOF h1))
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                  (hGetLine h2 >>= hPutStrLn h3 >>
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do h3 <- runSIO (... test ...)
   runSIO (... hPutStrLn h3 ...)
```

# Type-state

```
class Monadish m where
  return :: a -> m p p a
  (>>=)  :: m p q a -> (a -> m q r b) -> m p r b

test :: SHandle s 0 ->
        SIO s (1,[0]) (3,[2,0]) (SHandle s 2)
test h1 = do h2 <- hOpen "config" ReadMode
             fname <- hGetLine h2
             h3 <- hOpen fname WriteMode
             hPutStrLn h3 fname
             till (liftM2 (||) (hIsEOF h2) (hIsEOF h1))
                  (hGetLine h2 >>= hPutStrLn h3 >>
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do h3 <- runSIO (... test ...)
   runSIO (... hPutStrLn h3 ...)
```

# Waiter, there's a term in my type!

Rank-2 polymorphism (ab)used, for safety:

- mutable state `runST :: (∀s. ST s a) -> a`
- array index bounds
- environment classifiers
- staged lexical scope
- resource control
- automatic differentiation

Questions:

- How to reify type-level gensym?
- How to unify compile-time and run-time gensym?
- How to compare for equality?