transformation/optimization on representations used in theorem-proving tools that use extensions of the lambda-calculus as an underlying representation.

- **Failure to provide examples/applications that speak to a broader community:** Early work in partial evaluation often used the "power function", "dot product" or similar examples to illustrate a technique. Since PE concepts are now fairly well-understood in the PEPM community, such examples should be avoided in PEPM submissions and replaced with examples that could convey the utility of PE and other program transformation techniques to a larger audience. Our aim is to grow the number of people from other areas that look to PEPM for solutions relevant to their problems. People from other applications

San Francisco Chronicle

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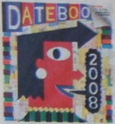

ART TOMORROW

Chronicle critics on what not to miss in theater, music, dance, art and film this year.

Datebook, 16

TRAGEDY AT YEAR'S END

A young father, knowing that gunfire was inevitable, threw himself over his 9-year-old daughter. In saving her, Albert Collins became San Francisco's final homicide victim in a deadly year.

Bay Area, B1

WUNDERKIND. OR NOT.

Rain keeps coming, snow pounds the Sierra — many homes, businesses will be without electricity for days because of storm's brutality

MORE THAN 50,000 STILL LACK POWER

An employee at Nick's Restaurant in Pacifica gets light from a propane lantern as she makes coffee. Like much of the city, Nick's operated without power as heavy rains and wind kept pounding the coast.

By John Wildermuth and Steve Rubenstein
Chronicle Staff Writers

The storm

CAMPAIGN 2008

This time, under-30 voters are showing up

By Joe Garofoli
Chronicle Staff Writer

For more than three decades, it was the hollowest of presidential campaign promises: "And we're going to get out the youth vote!" Yeah, right. Politicians rarely talked about the issues that mattered to young people, and the under-30 crowd returned the favor by not voting.

Young people did show up four years ago — to vote against George W. Bush. But on Thursday, they showed up in record numbers to vote *for* somebody, helping to propel Sen. Barack Obama to victory in the Iowa Democratic caucuses.

They showed that appealing to the under-30 vote doesn't have to be a hollow promise for candidates who know how to translate their online love into real-world votes.

The number of under-30 Iowa caucus-goers tripled compared with 2004, and more than 57 percent of young Democratic voters supported Obama. Exit polls found 22 percent of the nearly 239,000 Democratic voters were

▸ **Following the votes:** Independents are a major force this year. They comprise nearly half of New Hampshire's registered voters, and presidential hopefuls are scrambling for their attention as that state's primary approaches. A5

▸ **Democrats:** "Change" is the hot topic in a New Hampshire debate as a new poll shows Barack Obama and Hillary Rodham Clinton in a dead heat. A4

▸ **GOP:** High stakes kindle clashes among contenders for

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

$$\lambda x.\, x \times x \times x \times x \times x \times x \times x \times 1$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

$$\langle \lambda x.\, x \times x \times x \times x \times x \times x \times x \times 1 \rangle$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
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- special-purpose processors

$$\langle \lambda x.\ {\sim}(\mathtt{power}\ 7\ \langle x \rangle) \rangle$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

$$\texttt{run}\langle \lambda x.\ \sim(\mathrm{power}\ 7\ \langle x\rangle)\rangle$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

$$\mathtt{run}\langle \lambda x.\ \sim(\mathrm{power}\ 7\ \langle x\rangle)\rangle$$

$$\mathtt{run}\langle \lambda x.\ \sim(\mathrm{power}\ \langle x\rangle\ 7)\rangle \qquad \times$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

$$\mathtt{run}\langle \lambda x.\ \mathord{\sim}(\mathtt{power}\ 7\ \langle x\rangle)\rangle$$
$$\mathtt{run}\langle \lambda x.\ \mathord{\sim}(\mathtt{power}\ 7\ \langle 2\rangle)\rangle$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

$$\mathtt{run}\langle\lambda x.\ \mathtt{\sim}(\mathrm{power}\ 7\ \langle x\rangle)\rangle$$

$$\mathtt{run}\langle\lambda x.\ \mathtt{\sim}(\mathrm{power}\ 7\ \langle\mathtt{true}\rangle)\rangle \qquad \times$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
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Type safety

- generate well-typed, well-scoped code: no scope extrusion
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$$\mathtt{run}\,\langle \lambda x.\ {\sim}(\mathrm{power}\ 7\ \langle x\rangle)\rangle$$

$$\mathtt{run}\,\langle \lambda x.\ {\sim}(\mathrm{power}\ 7\ \langle \mathrm{true}\rangle)\rangle \qquad \times$$

$$\mathtt{run}\,\langle x\rangle \qquad \times$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
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- generate well-typed, well-scoped code: no scope extrusion
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$$\mathtt{run}\langle \lambda x.\ \sim(\mathrm{power}\ 7\ \langle x\rangle)\rangle$$

$$\mathtt{run}\langle \lambda x.\ \sim(\mathrm{power}\ 7\ \langle \mathtt{true}\rangle)\rangle \qquad \times$$

$$\mathtt{run}\langle x\rangle \qquad \times$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
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- generate well-typed, well-scoped code: no scope extrusion
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$$\mathtt{run}\langle\lambda x.\ \sim(\mathrm{power}\ 7\ \langle x\rangle)\rangle$$

$$\mathtt{run}\langle\lambda x.\ \sim(\mathrm{power}\ 7\ \langle\mathrm{true}\rangle)\rangle \qquad \times$$

$$\mathtt{run}\langle x\rangle \qquad \times$$

$$\mathtt{run}\langle\lambda x.\ \sim(\ldots\,\mathtt{run}\langle 2\rangle\,\ldots)\rangle$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

$\mathtt{run}\langle\lambda x.\ {\sim}(\mathrm{power}\ 7\ \langle x\rangle)\rangle$	
$\mathtt{run}\langle\lambda x.\ {\sim}(\mathrm{power}\ 7\ \langle\mathrm{true}\rangle)\rangle$	$\times$
$\mathtt{run}\langle x\rangle$	$\times$
$\mathtt{run}\langle\lambda x.\ {\sim}(\ldots\mathtt{run}\langle x\rangle\ldots)\rangle$	$\times$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

$$
\begin{array}{ll}
\mathtt{run}\,\langle\overbrace{\lambda x.}^{\text{gensym + binding}}\ \sim(\mathtt{power}\ 7\ \langle x\rangle)\rangle & \\
\mathtt{run}\,\langle\lambda x.\ \sim(\mathtt{power}\ 7\ \langle\mathtt{true}\rangle)\rangle & \times \\
\mathtt{run}\,\langle x\rangle & \times \\
\mathtt{run}\,\langle\lambda x.\ \sim(\ldots\mathtt{run}\,\langle x\rangle\ldots)\rangle & \times
\end{array}
$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

$$\mathtt{run}\langle \overbrace{\lambda y.}^{\text{gensym + binding}} \sim(\mathrm{power}\ 7\ \langle y \rangle)\rangle$$

$$\mathtt{run}\langle \lambda x. \sim(\mathrm{power}\ 7\ \langle \mathrm{true} \rangle)\rangle \qquad \times$$

$$\mathtt{run}\langle x \rangle \qquad \times$$

$$\mathtt{run}\langle \lambda x. \sim(\dots \mathtt{run}\langle x \rangle \dots)\rangle \qquad \times$$

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

Mutable state

- let insertion, assert insertion
- count generated operations

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

Mutable state and delimited control

- let insertion, assert insertion
- count generated operations
- partial evaluation of sum types and delimited control

Motivation: Typed staging with side effects

Code generation

- partial evaluation
- embedded domain-specific languages
- special-purpose processors

Type safety

- generate well-typed, well-scoped code: no scope extrusion
- splice open code yet run closed code: keep α-equivalence

Mutable state and delimited control

- let insertion, assert insertion
- count generated operations
- partial evaluation of sum types and delimited control

Pick two.

We translate staging away: Simplified MetaOCaml $\Rightarrow$ System F

Closing the stage

From staged code to typed closures

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Oleg Kiselyov
FNMOC
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Chung-chieh Shan
Rutgers University
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PEPM, January 7, 2008

?

let rec power n x =
    if $n = 0$
    then 1
    else $x \times$ (power $(n - 1)$ x)
let power7 = $\lambda x.$ (power 7 x)

let rec power n x =
    if $n = 0$
    then   1
    else $x \times$ (power $(n-1)$ x)
let power7 = $\lambda x.$ (power 7 x)

```
let rec power n c =
    if n = 0
    then ⟨1⟩
    else ⟨~c × ~(power (n − 1) c)⟩
let power7 = ⟨λx. ~(power 7 ⟨x⟩)⟩
```

From code to thunks

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle {\sim}c \times {\sim}(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ {\sim}(\text{power } 7\ \langle x \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda().\ 1 \\
\quad \text{else } \lambda().\ c\ () \times \text{power } (n-1)\ c\ () \\
\text{let power7} = \lambda().\ \lambda x.\ \text{power } 7\ (\lambda().\ x)\ ()
\end{array}
$$

From code to thunks

$$
\begin{array}{l}
\text{let rec power } n \ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle {\sim}c \times {\sim}(\text{power } (n-1) \ c) \rangle \\
\text{let power7} = \langle \lambda x. \ {\sim}(\text{power } 7 \ \langle x \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n \ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda(). \ 1 \\
\quad \text{else } \lambda(). \ c \ () \times \text{power } (n-1) \ c \ () \\
\text{let power7} = \lambda(). \ \lambda x. \ \text{power } 7 \ (\lambda(). \ x) \ ()
\end{array}
$$

From code to thunks

$$
\begin{array}{l}
\mathrm{let\ rec\ power}\ n\ c = \\
\quad \mathrm{if}\ n = 0 \\
\quad \mathrm{then}\ \langle 1 \rangle \\
\quad \mathrm{else}\ \langle \sim c \times \sim(\mathrm{power}\ (n-1)\ c) \rangle \\
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\end{array}
$$

$$\Downarrow$$

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\begin{array}{l}
\mathrm{let\ rec\ power}\ n\ c = \\
\quad \mathrm{if}\ n = 0 \\
\quad \mathrm{then}\ \lambda().\ 1 \\
\quad \mathrm{else}\qquad \lambda().\ c\ () \times \mathrm{power}\ (n-1)\ c\ () \\
\mathrm{let\ power7} = \qquad \lambda().\ \lambda x.\ \mathrm{power}\ 7\ (\lambda().\ x)\ ()
\end{array}
$$

From code to thunks

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle {\sim} c \times {\sim}(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ {\sim}(\text{power } 7\ \langle x \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda().\ 1 \\
\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda().\ c\ () \times v\ () \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda().\ x) \text{ in } \lambda().\ \lambda x.\ v\ ()
\end{array}
$$

From code to thunks

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
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$$\Downarrow$$

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\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda().\ 1 \\
\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda().\ c\ () \times v\ () \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda().\ x) \text{ in } \lambda().\ \lambda x.\ v\ ()
\end{array}
$$

From code to thunks

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle \sim c \times \sim(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ \sim(\text{power } 7\ \langle x \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda(\ ).\ 1 \\
\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda(\ ).\ c\ (\ ) \times v\ (\ ) \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda(\ ).\ x) \text{ in } \lambda().\ \lambda x.\ v\ (\ )
\end{array}
$$

From code to closures

$$
\begin{array}{l}
\text{let rec power } n \ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle \sim c \times \sim(\text{power } (n-1) \ c) \rangle \\
\text{let power7} = \langle \lambda x. \sim(\text{power } 7 \ \langle x \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n \ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda(x).\ 1 \\
\quad \text{else let } v = \text{power } (n-1) \ c \text{ in } \lambda(x).\ c\ (x) \times v\ (x) \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda(x).\ x) \text{ in } \lambda().\ \lambda x.\ v\ (x)
\end{array}
$$

From code to closures

$$\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle \sim c \times \sim(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ \sim(\text{power } 7\ \langle x \rangle) \rangle
\end{array}$$

$$\Downarrow$$

$$\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda(x).\ 1 \\
\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda(x).\ c\ (x) \times v\ (x) \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda(x).\ x) \text{ in } \lambda().\ \lambda x.\ v\ (x)
\end{array}$$

From code to closures

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle {\sim}c \times {\sim}(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ {\sim}(\text{power } 7\ \langle x \rangle) \rangle \\
\text{let power7sum} = \langle \lambda x.\ \lambda y.\ {\sim}(\text{power } 7\ \langle x + y \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda(x).\ 1 \\
\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda(x).\ c\ (x) \times v\ (x) \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda(x).\ x) \text{ in } \lambda().\ \lambda x.\ v\ (x)
\end{array}
$$

From code to closures

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
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\text{let power7sum} = \langle \lambda x.\ \lambda y.\ \sim(\text{power } 7\ \langle x + y \rangle) \rangle
\end{array}
$$

$$\Downarrow$$

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \lambda(x).\ 1 \\
\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda(x).\ c\ (x) \times v\ (x) \\
\text{let power7} = \text{let } v = \text{power } 7\ (\lambda(x).\ x) \text{ in } \lambda().\ \lambda x.\ v\ (x)
\end{array}
$$

From code to closures

$\text{let rec power } n\ c =$
$\quad \text{if } n = 0$
$\quad \text{then } \langle 1 \rangle$
$\quad \text{else } \langle {\sim}c \times {\sim}(\text{power } (n-1)\ c) \rangle$
$\text{let power7} = \langle \lambda x.\ {\sim}(\text{power } 7\ \langle x \rangle) \rangle$
$\text{let power7sum} = \langle \lambda x.\ \lambda y.\ {\sim}(\text{power } 7\ \langle x + y \rangle) \rangle$

$$\Downarrow$$

$\text{let rec power } n\ c =$
$\quad \text{if } n = 0$
$\quad \text{then } \lambda\ r\ .\ 1$
$\quad \text{else let } v = \text{power } (n-1)\ c \text{ in } \lambda\ r\ .\ c\ \ r\ \ \times v\ \ r$
$\text{let power7} = \text{let } v = \text{power } 7\ (\lambda(x).\ x) \text{ in } \lambda().\ \lambda x.\ v\ (x)$

From code to closures

let rec power n c =
  if $n = 0$
  then $\langle 1 \rangle$
  else $\langle \sim c \times \sim(\text{power } (n-1)\ c) \rangle$
let power7 = $\langle \lambda x.\ \sim(\text{power } 7\ \langle x \rangle) \rangle$
let power7sum = $\langle \lambda x.\ \lambda y.\ \sim(\text{power } 7\ \langle x + y \rangle) \rangle$

$\Downarrow$

let rec power n c =
  if $n = 0$
  then $\lambda\ r\ .\ 1$
  else let v = power $(n-1)$ c in $\lambda\ r\ .\ c\ \ r\ \times v\ \ r$
let power7 = let v = power 7 $(\lambda(x).\ x)$ in $\lambda().\ \lambda x.\ v\ (x)$
let power7sum = let v = power 7 $(\lambda(x,y).\ x + y)$ in $\lambda().\ \lambda x.\ v\ (x, y)$

From code to closures

$$
\begin{array}{l}
\text{let rec power } n \ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle \sim c \times \sim(\text{power } (n-1) \ c) \rangle \\
\text{let power7} = \langle \lambda x.\ \sim(\text{power } 7 \ \langle x \rangle) \rangle \\
\text{let power7sum} = \langle \lambda x.\ \lambda y.\ \sim(\text{power } 7 \ \langle x + y \rangle) \rangle \\
\text{let eta } f = \langle \lambda x.\ \sim(f \ \langle x \rangle) \rangle
\end{array}
$$

From higher-order code to higher-rank polymorphism

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle \sim c \times \sim(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ \sim(\text{power } 7\ \langle x \rangle) \rangle \\
\text{let power7sum} = \langle \lambda x.\ \lambda y.\ \sim(\text{power } 7\ \langle x + y \rangle) \rangle \\
\text{let eta } f = \langle \lambda x.\ \sim(f\ \langle x \rangle) \rangle
\end{array}
$$

$$
\begin{array}{l}
\text{let rec power}'\ n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else if } n \text{ mod } 2 = 0 \\
\qquad \text{then } \langle \text{let } z = \sim c \times \sim c \text{ in } \sim(\text{power}'\ (n \div 2)\ \langle z \rangle) \rangle \\
\qquad \text{else } \langle \text{let } z = \sim c \times \sim c \text{ in } \sim c \times \sim(\text{power}'\ ((n-1) \div 2)\ \langle z \rangle) \rangle
\end{array}
$$

From higher-order code to higher-rank polymorphism

$$
\begin{array}{l}
\text{let rec power } n\ c = \\
\quad \text{if } n = 0 \\
\quad \text{then } \langle 1 \rangle \\
\quad \text{else } \langle {\sim}c \times {\sim}(\text{power } (n-1)\ c) \rangle \\
\text{let power7} = \langle \lambda x.\ {\sim}(\text{power } 7\ \langle x \rangle) \rangle \\
\text{let power7sum} = \langle \lambda x.\ \lambda y.\ {\sim}(\text{power } 7\ \langle x + y \rangle) \rangle \\
\text{let eta } f = \langle \lambda x.\ {\sim}(f\ \langle x \rangle) \rangle
\end{array}
$$

$$\langle A \rangle \ \Rightarrow\ \ldots \to A$$

From higher-order code to higher-rank polymorphism

```
let rec power n c =
    if n = 0
    then ⟨1⟩
    else ⟨~c × ~(power (n − 1) c)⟩
let power7 = ⟨λx. ~(power 7 ⟨x⟩)⟩
let power7sum = ⟨λx. λy. ~(power 7 ⟨x + y⟩)⟩
let eta f = ⟨λx. ~(f ⟨x⟩)⟩
```

$$\begin{aligned} \langle A\rangle &\Rightarrow \ldots \to A \\ \langle A\rangle \to \langle B\rangle &\Rightarrow \forall\pi.\, ((\ldots,\pi)\to A)\to((\ldots,\pi)\to B) \end{aligned}$$

From higher-order code to higher-rank polymorphism

```
let rec power n c =
    if n = 0
    then ⟨1⟩
    else ⟨~c × ~(power (n – 1) c)⟩
let power7 = ⟨λx. ~(power 7 ⟨x⟩)⟩
let power7sum = ⟨λx. λy. ~(power 7 ⟨x + y⟩)⟩
let eta f = ⟨λx. ~(f ⟨x⟩)⟩
```

$$
\begin{aligned}
\langle A\rangle &\Rightarrow \ldots \to A \\
\langle A\rangle \to \langle B\rangle &\Rightarrow \forall\pi.\, ((\ldots,\pi) \to A) \to ((\ldots,\pi) \to B) \\
(\langle A\rangle \to \langle B\rangle) \to \langle C\rangle &\Rightarrow \forall\pi.\, (\forall\rho.\, ((\ldots,\pi,\rho) \to A) \\
&\qquad\qquad \to ((\ldots,\pi,\rho) \to B)) \\
&\qquad \to ((\ldots,\pi) \to C)
\end{aligned}
$$

Outline

Simplified MetaOCaml $\Rightarrow$ System F

Staged code $\Rightarrow$ Typed closures

Higher-order functions $\Rightarrow$ Higher-rank polymorphism

▶ Extension among environments $\Rightarrow$ Injection among types

Scope extrusion $\Rightarrow$ Type error

Coercions

$$\cdot \qquad \vdash$$

$$\Big\downarrow \text{Extend}$$

$$x \qquad \vdash$$

$$\Big\downarrow \text{Extend}$$

$$x, z_1 \qquad \vdash$$

$$\Big\downarrow \text{Extend}$$

$$x, z_1, z_2 \qquad \vdash$$

let rec power′ n c =
  if $n = 0$
  then $\langle 1 \rangle$
  else if n mod $2 = 0$
    then $\langle$let $z = {\sim}c \times {\sim}c$ in ${\sim}$(power′ (n ÷
    else $\langle$let $z = {\sim}c \times {\sim}c$ in ${\sim}c \times {\sim}$(power′

Coercions

$$c = \langle x \rangle$$

$$\Downarrow$$

$$
\begin{array}{ll}
\cdot & \vdash \\
\big\downarrow \text{Extend} & \\
x & \vdash \quad \lambda(x).\, x \\
\big\downarrow \text{Extend} & \\
x, z_1 & \vdash \\
\big\downarrow \text{Extend} & \\
x, z_1, z_2 & \vdash
\end{array}
$$

let rec power′ n c =
    if $n = 0$
    then $\langle 1 \rangle$
    else if n mod $2 = 0$
        then $\langle$let z = ~c × ~c in ~(power′ (n ÷
        else $\langle$let z = ~c × ~c in ~c × ~(power′

Coercions

$$
\begin{tikzcd}
 & & c = \langle x \rangle \arrow[d, Rightarrow] \\
\cdot \arrow[d, "\text{Extend}"] & \vdash & {} \\
x \arrow[d, "\text{Extend}"] & \vdash & \lambda(x).\, x \arrow[d, "\text{Coerce}"] \\
x, z_1 \arrow[d, "\text{Extend}"] & \vdash & \lambda(x, z_1).\, x \arrow[d, "\text{Coerce}"] \\
x, z_1, z_2 & \vdash & \lambda(x, z_1, z_2).\, x
\end{tikzcd}
$$

```
let rec power′ n c =
  if n = 0
  then ⟨1⟩
  else if n mod 2 = 0
       then ⟨let z = ~c × ~c in ~(power′ (n ÷
       else ⟨let z = ~c × ~c in ~c × ~(power′
```

Coercions

$$
\begin{array}{lcc}
 & & c = \langle x \rangle \\
 & & \Downarrow \\
\cdot & \vdash & \\
\Big\downarrow \text{Extend} & & \\
x & \vdash & \lambda(x).\, x \\
\Big\downarrow \text{Extend} & & \Big\downarrow \text{Coerce} \\
x, z_1 & \vdash & \lambda(x, z_1).\, x \\
\Big\downarrow \text{Extend} & & \Big\downarrow \text{Coerce} \\
x, z_1, z_2 & \vdash & \lambda(x, z_1, z_2).\, x
\end{array}
$$

$$
\begin{array}{c}
\mathrm{power}'\ n \\
\Downarrow \\
\lambda c.\, \lambda r. \ldots cr \times cr \ldots
\end{array}
$$

$\text{let rec power}'\ n\ c =$
$\quad \text{if } n = 0$
$\quad \text{then } \langle 1 \rangle$
$\quad \text{else if } n \bmod 2 = 0$
$\qquad \text{then } \langle \text{let } z = \sim c \times \sim c \text{ in } \sim(\mathrm{power}'\ (n \div$
$\qquad \text{else } \langle \text{let } z = \sim c \times \sim c \text{ in } \sim c \times \sim(\mathrm{power}'$

Coercions

$$
\begin{tikzcd}[column sep=small]
 & & c = \langle x \rangle \arrow[d, Rightarrow] & \mathtt{power}'\ n \arrow[d, Rightarrow] \\
\cdot \arrow[d, "\text{Extend}"] & \vdash & & \lambda c.\, \lambda r. \ldots cr \times cr \ldots \arrow[d, "\text{Coerce}"] \\
x \arrow[d, "\text{Extend}"] & \vdash & \lambda(x).\ x \arrow[d, "\text{Coerce}"] & \lambda c.\, \lambda(x,r). \ldots c(x,r) \times c(x,r) \ldots \arrow[d, "\text{Coerce}"] \\
x, z_1 \arrow[d, "\text{Extend}"] & \vdash & \lambda(x,z_1).\ x \arrow[d, "\text{Coerce}"] & \lambda c.\, \lambda(x,z_1,r). \ldots c(x,z_1,r) \times c(x,z_1,r) \ldots \arrow[d, "\text{Coerce}"] \\
x, z_1, z_2 & \vdash & \lambda(x,z_1,z_2).\ x & \lambda c.\, \lambda(x,z_1,z_2,r). \ldots c(x,z_1,z_2,r) \times c(x,z_1,z_2,r) \ldots
\end{tikzcd}
$$

Coercions

$$
\begin{tikzcd}[column sep=small]
 & & \langle \mathtt{int} \rangle \arrow[d, Rightarrow] & \langle \mathtt{int} \rangle \to \langle \mathtt{int} \rangle \arrow[d, Rightarrow] \\
\cdot \arrow[d, "\text{Extend}"] & \vdash & () \to \mathtt{int} \arrow[d, "\text{Coerce}"] & \forall \pi . ((\pi) \to \mathtt{int}) \to ((\pi) \to \mathtt{int}) \arrow[d, "\text{Coerce}"] \\
x \arrow[d, "\text{Extend}"] & \vdash & (\mathtt{int}) \to \mathtt{int} \arrow[d, "\text{Coerce}"] & \forall \pi . ((\mathtt{int}, \pi) \to \mathtt{int}) \to ((\mathtt{int}, \pi) \to \mathtt{int}) \arrow[d, "\text{Coerce}"] \\
x, z_1 \arrow[d, "\text{Extend}"] & \vdash & (\mathtt{int}, \mathtt{int}) \to \mathtt{int} \arrow[d, "\text{Coerce}"] & \forall \pi . ((\mathtt{int}, \mathtt{int}, \pi) \to \mathtt{int}) \to ((\mathtt{int}, \mathtt{int}, \pi) \to \mathtt{int}) \arrow[d, "\text{Coerce}"] \\
x, z_1, z_2 & \vdash & (\mathtt{int}, \mathtt{int}, \mathtt{int}) \to \mathtt{int} & \begin{array}{l} \forall \pi . ((\mathtt{int}, \mathtt{int}, \mathtt{int}, \pi) \to \mathtt{int}) \\ \quad \to ((\mathtt{int}, \mathtt{int}, \mathtt{int}, \pi) \to \mathtt{int}) \end{array}
\end{tikzcd}
$$

Coercions

$$\underbrace{\langle \mathtt{int} \rangle \qquad\qquad\qquad\qquad \langle \mathtt{int} \rangle \to \langle \mathtt{int} \rangle}_{\text{Indexed by source type (identity for int)}}$$

$$\left.\begin{array}{ll}
\cdot & \vdash \\
\Big\downarrow \text{Extend} & \\
x & \vdash \\
\Big\downarrow \text{Extend} & \\
x, z_1 & \vdash \\
\Big\downarrow \text{Extend} & \\
x, z_1, z_2 & \vdash
\end{array}\right\}$$

Induced by the extension
from the environment of creation
to the environment of use

We translate terms by induction on typing *derivations*, yet the translation is compositional in some senses.

Coercions elaborate environment polymorphism

In our source language

From environment classifiers (Taha, Nielsen, Calcagno, Moggi)

$$\langle \mathtt{int} \rangle^{\alpha}$$

to contextual modal type theory (Nanevski, Pfenning, Pientka)?

$$[\,]\,\mathtt{int} \quad [x : \mathtt{int}]\,\mathtt{int} \quad [x : \mathtt{int},\, z_1 : \mathtt{int}]\,\mathtt{int} \quad [x : \mathtt{int},\, z_1 : \mathtt{int},\, z_2 : \mathtt{int}]\,\mathtt{int}$$

Coercions elaborate environment polymorphism

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$[]\,\mathrm{int} \qquad [\mathrm{int}]\,\mathrm{int} \qquad [\mathrm{int}, \mathrm{int}]\,\mathrm{int} \qquad [\mathrm{int}, \mathrm{int}, \mathrm{int}]\,\mathrm{int}$

Our "de Bruijn indices" maintain α-equivalence and avoid the need for ρ-polymorphism and negative side conditions.

Coercions elaborate environment polymorphism

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$[\,]\,\mathrm{int} \qquad [\mathrm{int}]\,\mathrm{int} \qquad [\mathrm{int}, \mathrm{int}]\,\mathrm{int} \qquad [\mathrm{int}, \mathrm{int}, \mathrm{int}]\,\mathrm{int}$

Our "de Bruijn indices" maintain α-equivalence and avoid the need for ρ-polymorphism and negative side conditions.

In our target language

System F lacks environment polymorphism (weakening), so we roll our own.

Scope extrusion

How to count multiplications as we generate them?

Scope extrusion

How to count multiplications as we generate them?

$$
\begin{array}{l}
\mathsf{let\ count} = \mathsf{ref}\ 0 \\
\mathsf{let\ rec\ power}\ n\ c = \\
\quad \mathsf{if}\ n = 0 \\
\quad \mathsf{then}\ \langle 1 \rangle \\
\quad \mathsf{else}\ \mathsf{count} \leftarrow\ !\mathsf{count} + 1; \ldots
\end{array}
$$

Scope extrusion

How to count multiplications as we generate them?

Use environment may no longer extend creation environment.

int state is safe: the identity coercion is always available.

$$
\begin{array}{l}
\mathrm{let\ count} = \mathrm{ref}\ 0 \\
\mathrm{let\ rec\ power}\ n\ c = \\
\quad \mathrm{if}\ n = 0 \\
\quad \mathrm{then}\ \langle 1 \rangle \\
\quad \mathrm{else\ count} \leftarrow \mathrm{!count} + 1; \ldots
\end{array}
$$

$\langle \mathrm{int} \rangle$ state risks scope extrusion and running open code.

$$
\begin{array}{lll}
\mathrm{let}\ x = \mathrm{ref}\ \langle 1 \rangle\ \mathrm{in} & & \\
\langle \lambda y.\ \tilde{}(x \leftarrow \langle y \rangle; \langle () \rangle) \rangle; & & \\
!x & \rightsquigarrow & \langle y \rangle
\end{array}
$$

Conclusion

$$\begin{aligned}
\text{Simplified MetaOCaml} &\Rightarrow \text{System F}\\
\text{Staged code} &\Rightarrow \text{Typed closures}\\
\text{Higher-order functions} &\Rightarrow \text{Higher-rank polymorphism}\\
\text{Extension among environments} &\Rightarrow \text{Injection among types}\\
\text{Scope extrusion} &\Rightarrow \text{Type error}
\end{aligned}$$

Small-step operational semantics for source language
(need to show: preserved by translation)

Administrative reductions incur abstraction overhead
(eliminated by true staging) despite specialization