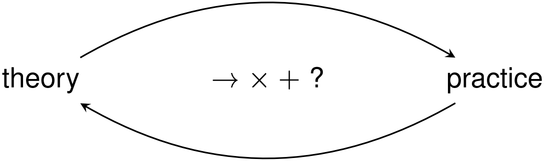
# What are these control hierarchies?

Chung-chieh Shan
Rutgers University

29 May 2011



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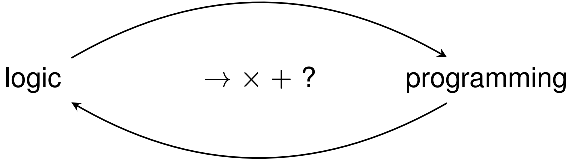


$$\alpha \to \beta$$

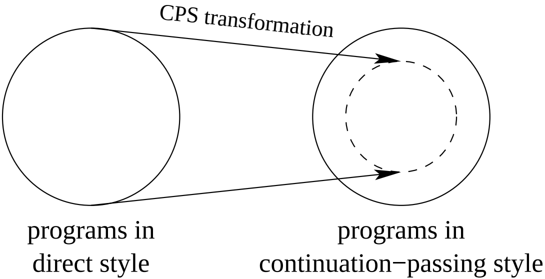
$\Big\downarrow$ CPS

$$\alpha \times (\beta \to \omega) \to \omega$$

$$\alpha \to \beta$$

$$\rightsquigarrow \text{CPS}$$

$$\alpha \times (\beta \to \omega) \to \omega$$




Danvy & Millikin

$$\alpha \to \beta$$

$$\Big\downarrow \text{CPS}$$

$$\alpha \times (\beta \to \omega) \to \omega$$

Logic guides codifying the pattern: Zeilberger

- Decompose positive vs negative expressions/variables
  function vs context connectives/constructions
- Decompose $\omega_1 \to \beta/\omega_2$ into $(\beta \multimap \omega_1) \triangleright \omega_2$
  $\alpha/\omega_1 \to \beta/\omega_2$ into $((\alpha \to \beta) \multimap \omega_1) \triangleright \omega_2$
- Negatives are polymorphic in the answer type;
  positives are specific in the answer type?

**Make nonsense impossible, common sense easy** (reflection)?

$$\alpha \to \beta$$

$$\Big\downarrow \text{CPS}$$

$$\alpha \times (\beta \to \omega) \to \omega$$

$$\Big\downarrow \text{CPS}$$

Danvy & Filinski

$$\alpha \times (\beta \times (\omega \to \varpi) \to \varpi) \times (\omega \to \varpi) \to \varpi$$

**Make nonsense impossible, common sense easy** (reflection)?

*What is the total population of the ten largest capitals in the US?*
Answering these types of complex questions compositionally involves first mapping the questions into logical forms (semantic parsing).

Liang, Jordan & Klein

*What is the total population of the ten largest capitals in the US?* Answering these types of complex questions compositionally involves first mapping the questions into logical forms (semantic parsing).

The filtering function $F$ rules out improperly-typed trees ...
To further reduce the search space ...

Think of DCS as a higher-level programming language tailored to natural language, which results in programs which are much simpler than the logically-equivalent lambda calculus formulae.

Liang, Jordan & Klein

# Alice knows Bob

$$\begin{aligned}\mathrm{Alice} &:: \mathrm{E}\\ \mathrm{Bob} &:: \mathrm{E}\\ \mathrm{know} &:: \mathrm{E} \to \mathrm{E} \to \mathrm{Bool}\end{aligned}$$

$$\mathrm{Alice} \mathbin{\text{₰}} (\mathrm{know} \mathbin{\$} \mathrm{Bob}) :: \mathrm{Bool}$$

# Alice knows Bob

$$
\begin{aligned}
\mathrm{Alice} &:: \mathbb{E} \\
\mathrm{Bob} &:: \mathbb{E} \\
\mathrm{know} &:: \mathbb{E} \to \mathbb{E} \to \mathrm{Bool}
\end{aligned}
$$

$$\mathrm{Alice} \;\reflectbox{\$}\; (\mathrm{know} \;\$\; \mathrm{Bob}) :: \mathrm{Bool}$$

$$
\dfrac{
  \dfrac{}{\mathbb{E}}\,\mathrm{Alice}
  \qquad
  \dfrac{
    \dfrac{}{\mathbb{E} \to \mathbb{E} \to \mathrm{Bool}}\,\mathrm{know}
    \qquad
    \dfrac{}{\mathbb{E}}\,\mathrm{Bob}
  }{\mathbb{E} \to \mathrm{Bool}}\,\$
}{\mathrm{Bool}}\,\reflectbox{\$}
$$

# Alice knows everyone

$$\texttt{Alice} :: \mathbb{E}$$
$$\texttt{Bob} \quad :: \mathbb{E}$$
$$\texttt{know} :: \mathbb{E} \to \mathbb{E} \to \text{Bool}$$

$$\texttt{type}\ \mathbf{M}\,\alpha = (\alpha \to \text{Bool}) \to \text{Bool}$$

$$\texttt{everyone} :: \mathbf{M}\,\mathbb{E}$$
$$\texttt{everyone}\ c = \text{all}\ c\ [\text{Alice}, \text{Bob}, ..]$$

Barker, de Groote, . . .

# Alice knows everyone

$$
\begin{aligned}
&\mathtt{Alice} :: \mathrm{E}\\
&\mathtt{Bob} \;\; :: \mathrm{E}\\
&\mathtt{know} :: \mathrm{E} \to \mathrm{E} \to \mathsf{Bool}
\end{aligned}
$$

$$\mathtt{type}\ \mathbf{M}\,\alpha = (\alpha \to \mathsf{Bool}) \to \mathsf{Bool}$$

$$
\begin{aligned}
&\mathtt{everyone} :: \mathbf{M}\,\mathrm{E}\\
&\mathtt{everyone}\ c = \mathsf{all}\ c\ [\mathsf{Alice}, \mathsf{Bob}, ..]
\end{aligned}
$$

$$
\dfrac{
  \dfrac{\dfrac{}{\mathrm{E}}\ \mathtt{Alice}}{\mathbf{M}\,\mathrm{E}}\ \mathtt{return}
  \qquad
  \dfrac{
    \dfrac{\dfrac{}{\mathrm{E} \to \mathrm{E} \to \mathsf{Bool}}\ \mathtt{know}}{\mathbf{M}(\mathrm{E} \to \mathrm{E} \to \mathsf{Bool})}\ \mathtt{return}
    \qquad
    \dfrac{}{\mathbf{M}\,\mathrm{E}}\ \mathtt{everyone}
  }{\mathbf{M}(\mathrm{E} \to \mathsf{Bool})}\ \mathsf{liftM2}\,(\$)
}{
  \dfrac{\mathbf{M}\,\mathsf{Bool}}{\mathsf{Bool}}\ (\$\ \mathsf{id})
}\ \mathsf{liftM2}\,(\reflectbox{\$})
$$

Barker, de Groote, ...

# Alice knows everyone

$$\begin{aligned}
\mathtt{Alice} &:: \mathrm{E} \\
\mathtt{Bob} &:: \mathrm{E} \\
\mathtt{know} &:: \mathrm{E} \to \mathrm{E} \to \mathsf{Bool}
\end{aligned}$$

$$\mathtt{type}\ \mathbf{M}\,\alpha = (\alpha \to \mathsf{Bool}) \to \mathsf{Bool}$$

$$\begin{aligned}
\mathtt{everyone} &:: \mathbf{M}\,\mathrm{E} \\
\mathtt{someone} &:: \mathbf{M}\,\mathrm{E} \\
\mathtt{most} &:: [\mathrm{E}] \to \mathbf{M}\,\mathrm{E}
\end{aligned}$$

Someone knows everyone

# Alice knows everyone

$$\begin{aligned}
&\texttt{Alice} :: \mathbb{E}\\
&\texttt{Bob} \;\; :: \mathbb{E}\\
&\texttt{know} :: \mathbb{E} \to \mathbb{E} \to \mathsf{Bool}
\end{aligned}$$

$$\texttt{type}\ \mathbf{M}\,\alpha = (\alpha \to \mathsf{Bool}) \to \mathsf{Bool}$$

$$\begin{aligned}
&\texttt{every} && :: [\mathbb{E}] \to \mathbf{M}\,\mathbb{E}\\
&\texttt{some} && :: [\mathbb{E}] \to \mathbf{M}\,\mathbb{E}\\
&\texttt{most} && :: [\mathbb{E}] \to \mathbf{M}\,\mathbb{E}\\
&\texttt{logician} && :: [\mathbb{E}]\\
&\texttt{programmer} && :: [\mathbb{E}]
\end{aligned}$$

Someone knows everyone
Most logicians know some programmer

# Alice knows everyone

$$\begin{aligned}
\mathtt{Alice} &:: \mathbb{E} \\
\mathtt{Bob} &:: \mathbb{E} \\
\mathtt{know} &:: \mathbb{E} \to \mathbb{E} \to \mathsf{Bool}
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\mathtt{some} &:: [\mathbb{E}] \to \mathbf{M}\,\mathbb{E} \\
\mathtt{most} &:: [\mathbb{E}] \to \mathbf{M}\,\mathbb{E} \\
\mathtt{logician} &:: [\mathbb{E}] \\
\mathtt{programmer} &:: [\mathbb{E}] \\
\mathtt{from} &:: [\mathbb{E}] \to \mathbb{E} \to [\mathbb{E}]
\end{aligned}$$

Someone knows everyone
Most logicians know some programmer
Most logicians know some programmer from Novi Sad

# Inverse scope: Someone knows everyone

$$
\dfrac{
  \dfrac{\dfrac{}{\mathbf{M}\,\mathrm{E}}\;\text{someone}
  \qquad
  \dfrac{
    \dfrac{\dfrac{}{\mathrm{E} \to \mathrm{E} \to \mathrm{Bool}}\;\text{know}}{\mathbf{M}(\mathrm{E} \to \mathrm{E} \to \mathrm{Bool})}\;\text{return}
    \qquad
    \dfrac{}{\mathbf{M}\,\mathrm{E}}\;\text{everyone}
  }{\mathbf{M}(\mathrm{E} \to \mathrm{Bool})}\;\text{liftM2}\,(\$)
  }{\mathbf{M}\,\mathrm{Bool}}\;\text{liftM2}\,(\reflectbox{\$})
}{\mathrm{Bool}}\;(\$\,\mathrm{id})
$$

# Inverse scope: Someone knows everyone

$$
\dfrac{\dfrac{\dfrac{}{\mathbf{M}\,\mathrm{E}}\,\text{someone} \qquad \dfrac{\dfrac{\dfrac{}{\mathrm{E}\to\mathrm{E}\to\mathrm{Bool}}\,\text{know}}{\mathbf{M}(\mathrm{E}\to\mathrm{E}\to\mathrm{Bool})}\,\text{return} \qquad \dfrac{}{\mathbf{M}\,\mathrm{E}}\,\text{everyone}}{\mathbf{M}(\mathrm{E}\to\mathrm{Bool})}\,\text{liftM2}\,(\$)}{\mathbf{M}\,\mathrm{Bool}}\,\text{liftM2}\,(\reflectbox{\$})}{\mathrm{Bool}}\,(\$\,\text{id})
$$

$$
\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{{\color{blue}\mathbf{M}}\mathrm{E}}\,\text{someone}}{{\color{red}\mathbf{M}}({\color{blue}\mathbf{M}}\mathrm{E})}\,\text{return} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\mathrm{E}\to\mathrm{E}\to\mathrm{Bool}}\,\text{know}}{\mathbf{M}(\mathrm{E}\to\mathrm{E}\to\mathrm{Bool})}\,\text{return}}{\mathbf{M}(\mathbf{M}(\mathrm{E}\to\mathrm{E}\to\mathrm{Bool}))}\,\text{return} \qquad \dfrac{\dfrac{}{{\color{red}\mathbf{M}}\mathrm{E}}\,\text{everyone}}{{\color{red}\mathbf{M}}({\color{blue}\mathbf{M}}\mathrm{E})}\,\text{liftM return}}{{\color{red}\mathbf{M}}({\color{blue}\mathbf{M}}(\mathrm{E}\to\mathrm{Bool}))}\,\text{liftM2}(\text{liftM2}\,(\$))}{{\color{red}\mathbf{M}}({\color{blue}\mathbf{M}}\mathrm{Bool})}\,\text{liftM2}(\text{liftM2}\,(\reflectbox{\$}))}{{\color{red}\mathbf{M}}\mathrm{Bool}}\,\text{liftM}(\$\,\text{id})}{\mathrm{Bool}}\,(\$\,\text{id})
$$

# Inverse linking: Combining hierarchies?

some programmer from Novi Sad
some programmer from every city May

$$\dfrac{\dfrac{}{[\mathrm{E}] \to \mathbf{M}\,\mathrm{E}}\,\text{some} \qquad \begin{array}{c}\dfrac{\dfrac{}{[\mathrm{E}] \to \mathbf{M}\,\mathrm{E}}\,\text{every} \quad \dfrac{}{[\mathrm{E}]}\,\text{city}}{\mathbf{M}\,\mathrm{E}}\,\$ \\ \vdots\ \text{programmer from} \\ \mathbf{M}[\mathrm{E}]\end{array}}{\mathbf{M}\,\mathrm{E}}\,???$$

# Inverse linking: Combining hierarchies?

some programmer from Novi Sad
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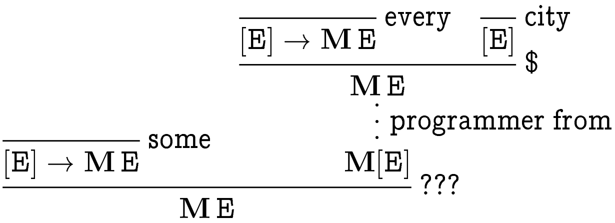


Someone knows everyone
Most logicians know some programmer
Most logicians know some programmer from every city

# From in-situ quantification to in-situ let-insertion

Write domain-specific code generators in multilevel languages

Nielson & Nielson, Taha

Continuations for code generation, especially let-insertion

Danvy & Filinski, Bondorf, Lawall & Danvy

$$\cdots \underline{+} \cdots \quad \rightsquigarrow \quad \underline{\texttt{let } t_1 = \cdots \texttt{ and } t_2 = \cdots \texttt{ and } t_3 = \cdots \texttt{ in } \cdots}$$

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$$\begin{aligned}(&\lambda e : \langle\alpha\rangle.\\ &\lambda c : \langle\alpha\rangle \to \langle\beta\rangle.\\ &\underline{\mathrm{let}\ x =}\ e\ \underline{\mathrm{in}}\ c\underline{x})\\ &\quad : \ \langle\alpha\rangle \to (\langle\alpha\rangle \to \langle\beta\rangle) \to \langle\beta\rangle\end{aligned}$$

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$$\begin{aligned}&(\lambda e : \langle\alpha\rangle^{\pi}.\\&\ \lambda c : \langle\alpha\rangle \to \langle\beta\rangle.\\&\ \underline{\text{let } x =}\ e\ \underline{\text{in}}\ c\underline{x})\\&\qquad : \ \langle\alpha\rangle \to (\langle\alpha\rangle \to \langle\beta\rangle) \to \langle\beta\rangle\end{aligned}$$

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$$\begin{aligned}
&(\lambda e : \langle\alpha\rangle^{\pi}.\\
&\ \lambda c : \forall\rho.\ \langle\alpha\rangle^{\pi,\rho} \to \langle\beta\rangle^{\pi,\rho}.\\
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$$\begin{aligned}&(\lambda e : \langle\alpha\rangle^{\pi,\sigma}.\\&\ \lambda c : \forall\rho.\ \langle\alpha\rangle^{\pi,\sigma,\rho} \to \langle\beta\rangle^{\pi,\sigma,\rho}.\\&\ \underline{\mathtt{let}\ x =}\ e\ \underline{\mathtt{in}}\ c\underline{x})\\&\qquad : \forall\sigma.\ \langle\alpha\rangle^{\pi,\sigma} \to (\forall\rho.\ \langle\alpha\rangle^{\pi,\sigma,\rho} \to \langle\beta\rangle^{\pi,\sigma,\rho}) \to \langle\beta\rangle^{\pi,\sigma}\end{aligned}$$

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&\qquad : \forall\sigma.\ \langle\alpha\rangle^{\pi,\sigma} \to (\forall\rho.\ \langle\alpha\rangle^{\pi,\sigma,\rho} \to \langle\beta\rangle^{\pi,\sigma,\rho}) \to \langle\beta\rangle^{\pi,\sigma}
\end{aligned}$$

Systematic translation, Kameyama, Kiselyov & Shan
but how does it fit CPS?
Want let-insertion at different scopes.

# Summary

Hierarchy 0: composing monad transformers
Hierarchy 1: composing monads (applicative functors)
Hierarchy 2: additional polymorphism at each level

**Make nonsense impossible, common sense easy?**