Lightweight static capabilities

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Goals

- Safety
  - no buffer overflow
  - modular arithmetic
- Performance (minimal runtime checking)
- Static assurance
- Available now
  - languages (Haskell, OCaml)
  - tools (compilers, debuggers)
  - features (IO, general recursion, mutation)
  - algorithms (Knuth-Morris-Pratt string search)

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```
bsearch cmp arr key = brand arr (\arr ->
  let rec loop i k = compare i k None (\i' k' ->
    let j = middle i' k' and x = get arr j in
    case cmp x key of LT -> loop (succ j) k
                      EQ -> Just (unbi j, x)
                      GT -> loop i (pred j))
  in loop)
```

Static capabilities

Continuum of correctness

- Assure safety properties, not full correctness
- Extend trust from small kernel to large sandbox

System requirements

- Higher-rank polymorphism
- Phantom types instead of dependent types

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System requirements

- Higher-rank polymorphism
- Phantom types instead of dependent types

A *capability* authorizes access to a protected object and certifies that a safety condition holds.

Outline

Trivial example: Empty-list checking

- List reverse
- Abstract data type to witness a runtime invariant
- Formalization: putting data constructors to work

Main example: Array-bounds checking

- Binary search
- Higher-rank polymorphism for an infinite family of invariants
- Formalization: lightweight dependent typing

List reverse

Starting point: ensure safety by *redundant* runtime checks.

```
rev l acc = if null l then acc
            else rev (tail l) (cons (head l) acc)
```

Idea: record the result of `null` check by wrapping the list type.

```
newtype List+ a = Nonempty (List a)
```

Runtime check supplies certifying witness to continuation.

```
indeed :: List a -> w -> (List+ a -> w)
head   :: List+ a -> a
tail   :: List+ a -> List a
```

Now `head` and `tail` need not check safety—

```
rev l acc = indeed l acc
            (\l -> rev (tail l) (cons (head l) acc))
```

—as long as `Nonempty` does not wrap an empty list.

Abstract data type to witness a runtime invariant

Use Milner's idea in LCF/ML:

- Divide the program into a small *security kernel* and a large *client sandbox*.

```
module Kernel
(
    List, List+,
    nil, cons,
    indeed, head, tail
)
where ...
```

- Using a module or namespace system, ensure that only the security kernel may apply the data constructor `Nonempty`.

Formalize security as an invariant of an abstract data type.

System F

Metavariables

Term variables	x, y, z
Terms	E
Type variables	s, t
Types	N, T, W
Natural numbers	m, n

System F

Metavariables

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Type variables	s, t
Types	N, T, W
Natural numbers	m, n

$$\frac{T : \star \quad T' : \star}{T \to T' : \star} \qquad \frac{\begin{array}{c}[t : \star] \\ \vdots \\ T' : \star\end{array}}{\forall t. T' : \star}$$

$$\frac{T : \star \quad \begin{array}{c}[x : T] \\ \vdots \\ E : T'\end{array}}{\lambda x. E : T \to T'} \qquad \frac{E_1 : T \to T' \quad E_2 : T}{E_1 E_2 : T'} \qquad \frac{\begin{array}{c}[t : \star] \\ \vdots \\ E : T'\end{array}}{\Lambda t. E : \forall t. T'} \qquad \frac{E : \forall t. T' \quad T : \star}{E T : T' \{t \mapsto T\}}$$

Putting data constructors to work

$$\frac{T : \star}{\mathrm{List}\, T : \star} \qquad \frac{T : \star}{\mathrm{List}^{+}\, T : \star} \qquad \frac{}{\mathrm{Int} : \star}$$

$$\frac{T : \star}{\mathrm{nil} : \mathrm{List}\, T} \qquad \frac{E_1 : T \quad E_2 : \mathrm{List}\, T}{E_1 :: E_2 : \mathrm{List}\, T} \qquad \frac{E_1 : T \quad E_2 : \mathrm{List}\, T}{\mathrm{nonempty}\,(E_1 :: E_2) : \mathrm{List}^{+}\, T} \qquad \frac{}{n : \mathrm{Int}}$$

$$\frac{E : \mathrm{List}\, T \quad E_1 : W \quad E_2 : \mathrm{List}^{+}\, T \to W}{\mathrm{indeed}\, E\, E_1\, E_2 : W} \qquad \frac{E : \mathrm{List}^{+}\, T}{\mathrm{head}\, E : T} \qquad \frac{E : \mathrm{List}^{+}\, T}{\mathrm{tail}\, E : \mathrm{List}\, T}$$

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$$\frac{T : \star}{\mathrm{List}\, T : \star} \qquad \frac{T : \star}{\mathrm{List}^+\, T : \star} \qquad \frac{}{\mathrm{Int} : \star}$$

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Small-step operational semantics

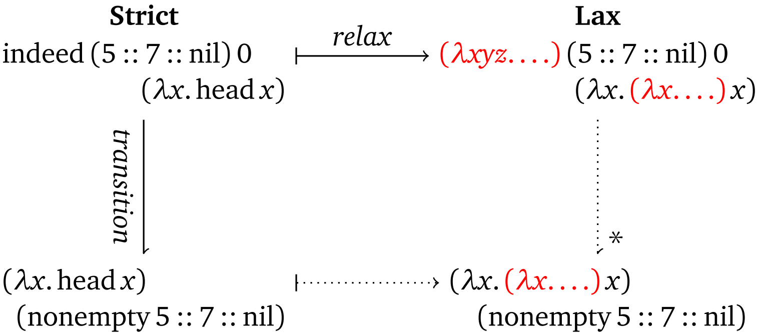
$$
\dfrac{
\begin{array}{c}\vdots\\ 5 :: 7 :: \mathrm{nil} : \mathrm{List\,Int}\end{array}
\qquad
\dfrac{}{0 : \mathrm{Int}}
\qquad
\begin{array}{c}\vdots\\ \lambda x.\,\mathrm{head}\,x : \mathrm{List}^{+}\,\mathrm{Int} \to \mathrm{Int}\end{array}
}{
\mathrm{indeed}\,(5 :: 7 :: \mathrm{nil})\,0\,(\lambda x.\,\mathrm{head}\,x) : \mathrm{Int}
}
$$

$$\Big\downarrow \ \textit{transition}$$

$$
\dfrac{
\begin{array}{c}\vdots\\ \lambda x.\,\mathrm{head}\,x : \mathrm{List}^{+}\,\mathrm{Int} \to \mathrm{Int}\end{array}
\qquad
\dfrac{
\dfrac{}{5 : \mathrm{Int}}
\qquad
\begin{array}{c}\vdots\\ 7 :: \mathrm{nil} : \mathrm{List\,Int}\end{array}
}{
\mathrm{nonempty}\,(5 :: 7 :: \mathrm{nil}) : \mathrm{List}^{+}\,\mathrm{Int}
}
}{
(\lambda x.\,\mathrm{head}\,x)\big(\mathrm{nonempty}\,(5 :: 7 :: \mathrm{nil})\big) : \mathrm{Int}
}
$$

Formalization

Small-step semantics (⇂) with syntax-directed translation (↦)



Formalization

Small-step semantics ($\downarrow$) with syntax-directed translation ($\mapsto$)

Strict		**Lax**
$\text{indeed}\,(5 :: 7 :: \text{nil})\,0\,(\lambda x.\,\text{head}\,x)$	$\xmapsto{\textit{relax}}$	$(\lambda xyz.\ldots)\,(5 :: 7 :: \text{nil})\,0\,(\lambda x.\,(\lambda x.\ldots)\,x)$
$\downarrow$ *transition*		$\downarrow^{*}$
$(\lambda x.\,\text{head}\,x)\,(\text{nonempty}\,5 :: 7 :: \text{nil})$	$\mapsto$	$(\lambda x.\,(\lambda x.\ldots)\,x)\,(\text{nonempty}\,5 :: 7 :: \text{nil})$

Relaxation preserves typing, valuehood, and transitions*. To prove:

- The kernel is implemented in Lax as specified in Strict.
- The sandbox constructs are identical in Lax and in Strict.

Formalization

Small-step semantics ($\rightharpoondown$) with syntax-directed translation ($\mapsto$)

$$\begin{array}{ccc} \textbf{Strict} & & \textbf{Lax} \\ \text{indeed}\,(5 :: 7 :: \text{nil})\,0 & \xmapsto{\textit{relax}} & (\lambda xyz.\ldots)\,(5 :: 7 :: \text{nil})\,0 \\ (\lambda x.\,\text{head}\,x) & & (\lambda x.\,(\lambda x.\ldots)\,x) \end{array}$$

We call a Lax program *sandboxed* if it uses kernel constructs only by inlining the kernel implementation.

Extend trust from kernel to sandbox

- Relaxation preserves typing, valuehood, and transitions*.
- Every (well-typed) sandboxed Lax program is the relaxation of some (well-typed) Strict program.
- Strict enjoys progress and preservation: well-typed Strict code does not go wrong.

Hence, well-typed sandboxed Lax code does not go wrong.

Outline

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Main example: Array-bounds checking

- Binary search
- Higher-rank polymorphism for an infinite family of invariants
- Formalization: lightweight dependent typing

Lightweight dependent typing

$$\frac{}{\bar{n} : \star} \qquad \frac{N : \star \quad T : \star}{\mathrm{List}^N\, T : \star} \qquad \frac{N : \star}{\mathrm{Int}^N : \star} \qquad \frac{N : \star}{\mathrm{Int}^N_{\mathrm{L}} : \star} \qquad \frac{N : \star}{\mathrm{Int}^N_{\mathrm{H}} : \star}$$

$$\frac{E_1 : T \quad \ldots \quad E_n : T}{\mathrm{array}\, E_1 :: \ldots E_n :: \mathrm{nil} : \mathrm{List}^{\bar{n}}\, T} \qquad \frac{1 \leq m \leq n}{m_{\mathrm{I}} : \mathrm{Int}^{\bar{n}}} \qquad \frac{1 \leq m}{m_{\mathrm{L}} : \mathrm{Int}^{\bar{n}}_{\mathrm{L}}} \qquad \frac{m \leq n}{m_{\mathrm{H}} : \mathrm{Int}^{\bar{n}}_{\mathrm{H}}}$$

$$\frac{E : \mathrm{List}\, T \quad E' : \forall s.\, \mathrm{List}^s\, T \to \mathrm{Int}^s_{\mathrm{L}} \to \mathrm{Int}^s_{\mathrm{H}} \to W}{\mathrm{brand}\, E\, E' : W} \qquad \frac{E_1 : \mathrm{List}^N\, T \quad E_2 : \mathrm{Int}^N}{\mathrm{get}\, E_1\, E_2 : T}$$

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Small-step operational semantics

$$\dfrac{\dfrac{\vdots}{5 :: 7 :: \mathrm{nil} : \mathrm{List\,Int}} \qquad \dfrac{\vdots}{\begin{array}{r}\Lambda s.\,\lambda xyz.\,\mathrm{compare}\,y\,z\,0\,\lambda yz.\,\mathrm{get}\,x\,(\mathrm{middle}\,y\,z)\\ : \forall s.\,\mathrm{List}^s\,\mathrm{Int} \to \mathrm{Int}^s_{\mathrm{L}} \to \mathrm{Int}^s_{\mathrm{H}} \to \mathrm{Int}\end{array}}}{\mathrm{brand}\,(5 :: 7 :: \mathrm{nil})\;\Lambda s.\,\lambda xyz.\,\mathrm{compare}\,y\,z\,0\,\lambda yz.\,\mathrm{get}\,x\,(\mathrm{middle}\,y\,z) : \mathrm{Int}}$$

$$\downarrow$$

$$\big(\Lambda s.\,\lambda xyz.\,\mathrm{compare}\,y\,z\,0\,\lambda yz.\,\mathrm{get}\,x\,(\mathrm{middle}\,y\,z)\big)\;\bar{2}\;(\mathrm{array}\,5 :: 7 :: \mathrm{nil})\;1_{\mathrm{L}}\;2_{\mathrm{H}}$$

$$\downarrow_{*}$$

$$\dfrac{\dfrac{\vdots}{\mathrm{array}\,5 :: 7 :: \mathrm{nil} : \mathrm{List}^{\bar{2}}\,\mathrm{Int}} \qquad \dfrac{\overline{1 \le 1 \le 2}}{1_{\mathrm{I}} : \mathrm{Int}^{\bar{2}}}}{\mathrm{get}\,(\mathrm{array}\,5 :: 7 :: \mathrm{nil})\;1_{\mathrm{I}} : \mathrm{Int}}$$

Lightweight dependent typing

$$\frac{}{\bar{n} : \star} \qquad \frac{N : \star \quad T : \star}{\mathrm{List}^N\, T : \star} \qquad \frac{N : \star}{\mathrm{Int}^N : \star} \qquad \frac{N : \star}{\mathrm{Int}^N_{\mathrm{L}} : \star} \qquad \frac{N : \star}{\mathrm{Int}^N_{\mathrm{H}} : \star}$$

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dependent types (Martin-Löf, Dybjer, . . .)
singleton types (Hayashi, Xi, Stone, . . .)
phantom types (. . . , Fluet & Pucella, . . .)
reflecting values through types (Thurston, Kiselyov & Shan, . . .)

$$\mathrm{brand}\, E\, E' : W \qquad\qquad \mathrm{get}\, E_1\, E_2 : T$$

$$\frac{E_{\mathrm{L}} : \mathrm{Int}^{N}_{\mathrm{L}} \quad E_{\mathrm{H}} : \mathrm{Int}^{N}_{\mathrm{H}} \quad E_1 : W \quad E_2 : \mathrm{Int}^{N} \to \mathrm{Int}^{N} \to W}{\mathrm{compare}\, E_{\mathrm{L}}\, E_{\mathrm{H}}\, E_1\, E_2 : W}$$

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Putting more data constructors to work

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Memory locations as term constants (Morrisett et al., Moggi & Sabry)

$$\mathrm{brand}\, E\, E' : W \qquad \mathrm{get}\, E_1\, E_2 : T$$

$$\frac{E_{\mathrm{L}} : \mathrm{Int}^N_{\mathrm{L}} \quad E_{\mathrm{H}} : \mathrm{Int}^N_{\mathrm{H}} \quad E_1 : W \quad E_2 : \mathrm{Int}^N \to \mathrm{Int}^N \to W}{\mathrm{compare}\, E_{\mathrm{L}}\, E_{\mathrm{H}}\, E_1\, E_2 : W}$$

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Nonces in security protocols

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Ill-typed term: brand $(5 :: 7 :: \mathrm{nil})\, \Lambda s.\, \lambda xyz.\, \mathrm{get}\, x\, 1_{\mathrm{I}}$ $\quad \mathrm{t}^N \to W$

(Can’t open branded lock with unbranded key)

$$\frac{E_1 : \mathrm{Int}^N \quad E_2 : \mathrm{Int}^N}{\mathrm{middle}\, E_1\, E_2 : \mathrm{Int}^N} \qquad \frac{E : \mathrm{Int}^N}{\mathrm{succ}\, E : \mathrm{Int}^N_{\mathrm{L}}} \qquad \frac{E : \mathrm{Int}^N}{\mathrm{pred}\, E : \mathrm{Int}^N_{\mathrm{H}}} \qquad \frac{E : \mathrm{Int}^N}{\mathrm{unbi}\, E : \mathrm{Int}}$$

Rights amplification

$$\frac{}{\bar{n} : \star} \qquad \frac{N : \star \quad T : \star}{\text{List}^N\, T : \star} \qquad \frac{N : \star}{\text{Int}^N : \star} \qquad \frac{N : \star}{\text{Int}^N_{\text{L}} : \star} \qquad \frac{N : \star}{\text{Int}^N_{\text{H}} : \star}$$

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> With rights amplification, the authority accessible from bringing two references together can exceed the sum of authorities provided by each individually. The classic example is the can and the can-opener—only by bringing the two together do we obtain the food in the can.
>
> —Miller et al.

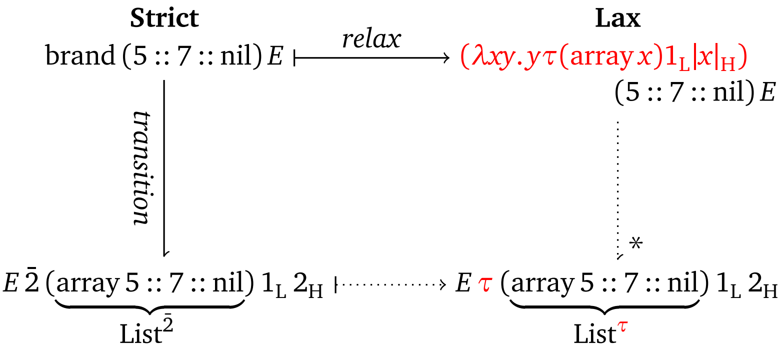
$$\frac{E_1 : \text{List}^{N}\, T \quad E_2 : \text{Int}^{N}}{\text{get}\, E_1\, E_2 : T}$$

$$\frac{\dots\text{nt}^N \to \text{Int}^N \to W}{W}$$

$$\text{middle}\, E_1\, E_2 : \text{Int}\dots \qquad \text{succ}\, E : \text{Int}_{\text{L}}\dots \qquad \frac{\dots : \text{Int}^N}{\text{pred}\, E : \text{Int}^N_{\text{H}}} \qquad \frac{E : \text{Int}^N}{\text{unbi}\, E : \text{Int}}$$

Formalization

Small-step semantics ($\downharpoonright$) with syntax-directed translation ($\mapsto$)



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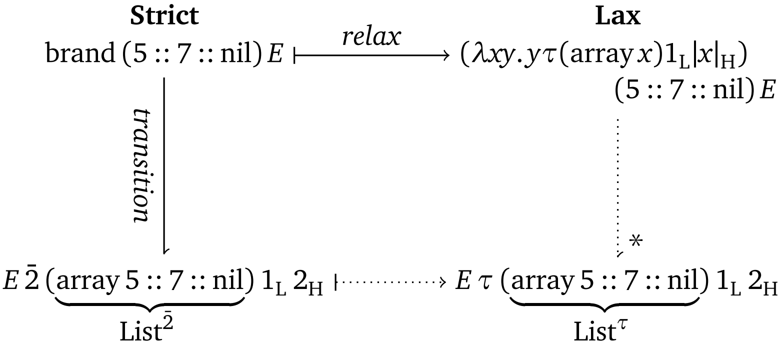

Relaxation preserves typing, valuehood, and transitions*. To prove:

- The kernel is implemented in Lax as specified in Strict.
- The sandbox constructs are identical in Lax and in Strict.

Formalization

Small-step semantics ($\downharpoonleft$) with syntax-directed translation ($\mapsto$)

$$\begin{array}{ccc} \textbf{Strict} & & \textbf{Lax} \\ \text{brand}\,(5::7::\text{nil})\,E & \xmapsto{\;\textit{relax}\;} & (\lambda xy.\,y\,\tau(\text{array}\,x)1_{\text{L}}|x|_{\text{H}}) \\ & & (5::7::\text{nil})\,E \end{array}$$

We call a Lax program *sandboxed* if it uses kernel constructs only by inlining the kernel implementation.

Extend trust from kernel to sandbox

- Relaxation preserves typing, valuehood, and transitions*.
- Every (well-typed) sandboxed Lax program is the relaxation of some (well-typed) Strict program.
- Strict enjoys progress and preservation: well-typed Strict code does not go wrong.

Hence, well-typed sandboxed Lax code does not go wrong.

Twelf mechanization

Theorem (Progress)

Every well-typed term either is a value or transitions to another term.

Proof.

By induction on evaluation contexts. □

Twelf mechanization

Lemma

A value never has any type of the form $\mathrm{Int}^{T \to T'}$.

Proof.

There is only one case, the value nil. □

Theorem (Progress)

Every well-typed term either is a value or transitions to another term.

Proof.

By induction on evaluation contexts. □

Twelf mechanization

Lemma

The expression nil *never has any type of the form* Int^{T}.

Proof.

There are no cases. □

Lemma

A value never has any type of the form $\mathrm{Int}^{T \to T'}$.

Proof.

There is only one case, the value nil. □

Theorem (Progress)

Every well-typed term either is a value or transitions to another term.

Proof.

By induction on evaluation contexts. □

Summary

A concrete, rigorous, practical framework for extending trust from a small kernel to a large program

Available now

- Type proxies for values, instead of dependent types
- Download all code online, with more substantial examples
  - Folding over multiple arrays of various sizes
  - Knuth-Morris-Pratt string search

Easy to verify small kernel

- Prove progress and preservation for specification
- Prove implementation corresponds to specification

Ongoing work

- More examples (any suggestions?)
- Characterize lightweight dependent typing