

# Scale-Degree Function: Cognition Research and Its Application to Aural-Skills Pedagogy

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The Center for Research on Concepts and Cognition (CRCC) at Indiana University (like its predecessor, the Fluid Analogies Research Group (FARG) at the University of Michigan), under the direction of Douglas Hofstadter, studies creativity in human cognition through computer models of “conceptual slippage” in analogical thinking and through reflective experimentation in “creative microdomains”. Developing an analogous model of music cognition suggests viewing “scale-degree function” as an emergent property of the musical concepts defined below. This view of scale-degree function draws on current research in music cognition as well as applications of Gestalt psychology to the perception of art and it has clear ramifications for the pedagogy of aural skills.

### *Internal representations of musical relationships*

The terms “audiate”, “hear as”, and “trace” are important terms for the internal representation of musical relationships. The term “audiate” means to hear internally sounds that are not physically present [Gordon 1988]. As used in this paper, the phrase “hear as” means to give meaning to an audiated sound by (subconsciously) assigning it to a category, and the term “trace” means the internal representation of a note that is still melodically active. (These and other terms defined in this paper are summarized in Example 1.)

These terms allow us to distinguish between “stable” and “unstable” notes. To hear a note as unstable means to audiate a more stable note to which it tends to move and a path (usually involving step-wise motion) that would take it there, displacing its trace.

The distinctions consonant/dissonant and stable/unstable should not be confused. The terms “consonant and dissonant” depend on notation, while the terms “stable and unstable” refer to experience. The terms “consonant and dissonant” depend on interval names, while the terms “stable and unstable” may be used to describe the perceived tendencies of individual notes (or of harmonies) in context. The terms “consonant and dissonant” create mutually exclusive categories, while the terms “stable and unstable” can be used only comparatively. Graybill [1987] has commented on the importance for theory instruction of making the differences between these distinctions clear. Butler [1992] makes a similar distinctions between “sensory consonance and dissonance” and “musical consonance and dissonance”.

**Example 1: Table of definitions.**

**INTERNAL REPRESENTATION**

**audiate.** To hear internally sounds that are not physically present [Gordon 1988].

**hear as.** To give meaning to an audiated sound by (subconsciously) assigning it to a category.

**trace.** The internal representation of a note that is still melodically active.

**unstable.** To hear a note as unstable means to audiate a more stable note to which it tends to move and a path that would take it there, displacing its trace.

**STEPS AND LEAPS**

**step.** Half steps and whole steps. The second note in a melodic step tends to displace the trace of the first.

**step collection.** A collection of pitches that can be ordered in an ascending list so that (1) all pitches that are adjacent in the list are a step apart and (2) no two pitches that are not adjacent in the list are a step apart.

**step-collection class.** A class of step collections that are equivalent under transposition.

**leap.** Thirds and larger intervals. The second note in a melodic leap does not tend to displace the trace of the first.

**leap collection.** A set of pitches in which no two pitches are a step apart.

**leap-collection class.** A class of leap collections that are equivalent under transposition.

**proper.** A step collection in which no two pitches—nor any of their octave equivalents—that are not adjacent in the list (except the first and last) are a step apart, or a leap collection in which no two pitches—nor any of their octave equivalents—are a step apart.

**closed.** A proper step collection in which the pitch an octave above the first note in the list is a step above the last note in the collection.

**open.** A step collection in which the pitch an octave above the first note in the list is not a step above the last note in the collection.

**incomplete.** A proper step collection to which a note can be added to produce another proper step collection, or a proper leap collection to which a pitch can be added to produce another proper leap collection.

**complete.** A proper step collection to which no note can be added to produce another proper step collection, or a proper leap collection to which no note can be added to produce another proper leap collection.

**MUSICAL FORCES**

**gravity.** The tendency of an unstable note to descend to a more stable note.

**magnetism.** The tendency of an unstable note to move (up or down) to the nearest stable pitch. (Magnetism is affected by distance—the closer we get to a goal, the more it attracts us.)

**inertia.** The tendency of a pattern of musical motion to continue in the same direction.

**pattern completion.** The tendency of a pattern of musical motion to continue until it creates a stable shape.

### *Steps and leaps*

This definition of “unstable” suggests that intervals may be divided into two classes: steps and leaps. Augmented unisons, minor seconds, and major seconds are called steps. (A diminished third is a step whose name suggests that it is heard as a leap.) Minor thirds and major thirds are called small leaps. (An augmented second is a leap whose name suggests that it is heard as a step.) Fourths and larger intervals are called large leaps.

In a melodic step, the second note tends to replace the “trace” of the first, leaving one trace in musical memory. In a melodic leap, the second note tends to support the trace of the first, leaving two traces in musical memory. Experiments in music perception tend to support this distinction between steps and leaps [Miller and Heise 1950; Van Noorden 1975; Kubovy 1981]. Published discussions in music cognition [e.g., Bharucha 1984; Deutsch 1982b; and Dowling 1973] often seem to depend implicitly on this distinction. What Hindemith called a “step-progression” seems analogous to what, in visual perception, the Gestalt psychologists called the “phi phenomenon”. And the theoretical discussions of Westergaard [1975], Larson [1987], and Dembski [1988] build explicitly upon this distinction.

The perceptual importance of the step/leap distinction to music cognition leads to the following definitions of “step collections”, “leap collections”, and “frames”.

### *Step collections*

A “step collection” is a special set of pitches. (A step collection is not a set of pitch classes—the register of the pitches is important.) The pitches in a step collection can be ordered in an ascending list so that (1) all pitches that are adjacent in the list are a step apart and (2) no two pitches that are not adjacent in the list are a step apart. The second condition can be modified slightly to produce a third condition, true of all “proper” step collections: (3) no two pitches—nor any of their octave equivalents—that are not adjacent in the list (except the first and last) are a step apart. The first condition ensures that the collection can be heard as a complete filling in of a musical space (this follows from our recognition that melodic leaps tend to leave the “trace” of a note “hanging” in our musical memories). The second condition ensures that no note will be heard as redundant in the filling of that space (this also reflects our desire to avoid confusion and the fact that either a whole step or half step can be heard as a step). The third condition ensures that the first and last pitches are less than an octave apart and that adding octave equivalents to a proper step collection will result in a step collection.

The step collections may be placed in “step-collection classes”—classes of equivalence under transposition (for reasons that will become clear, these classes are not equivalent under inversion). Examples 2 and 3 (on pages 4 and 5) list all proper step-collection classes. The first column of both examples lists pitches by their integer notation. The second column lists them by letter names. The third column lists the intervals (in half steps) formed by adjacent notes in the step collection (the final number in this list, in parentheses, indicates the interval between the last pitch in the list and the pitch an octave higher than the first pitch in the list).

If no pitch can be added to a proper step collection to produce another proper step collection, then it is called “complete”. If a pitch can be added to a proper step collection to produce another proper step collection, then it is called “incomplete”. If the note an octave above the first note in the list is a step above the last note in the list of a proper step collection, then it is called “closed”. If the note an octave above the first note in the list is not a step above the last note in the list of a proper step collection, then it is called “open”. All complete collections are closed and all closed collections are complete. All incomplete collections are open, but not all open collections are incomplete.

**Example 2: A list of the proper open step-collection classes.**

<i>pitch integers</i>	<i>letter names</i>	<i>adjacent intervals</i>	<i>complete?</i>	<i>closed?</i>	<i>Forte name</i>	<i>vector</i>
0 1	CDb	1 (11)	incomplete	open		100000
0 2	CD	2 (10)	incomplete	open		010000
0 1 3	CDbEb	1 2 (9)	incomplete	open	3-2	111000
0 2 3	CDEb	2 1 (9)	incomplete	open	3-2*	111000
0 2 4	CDE	2 2 (8)	incomplete	open	3-6	020100
0 1 3 4	CDbEbE	1 2 1 (8)	incomplete	open	4-3	212100
0 1 3 5	CDbEbF	1 2 2 (7)	incomplete	open	4-11	121110
0 2 3 5	CDEbF	2 1 2 (7)	incomplete	open	4-10	122010
0 2 4 5	CDEF	2 2 1 (7)	incomplete	open	4-11*	121110
0 2 4 6	CDEF#	2 2 2 (6)	incomplete	open	4-21	030201
0 1 3 4 6	CDbEbEF#	1 2 1 2 (6)	incomplete	open	5-10	223111
0 1 3 5 6	CDbEbFGb	1 2 2 1 (6)	incomplete	open	5-Z12	222121
0 1 3 5 7	CDbEbFG	1 2 2 2 (5)	incomplete	open	5-24	131221
0 2 3 5 6	CDEbFGb	2 1 2 1 (6)	incomplete	open	5-10*	223111
0 2 3 5 7	CDEbFG	2 1 2 2 (5)	incomplete	open	5-23	132130
0 2 4 5 7	CDEFG	2 2 1 2 (5)	incomplete	open	5-23*	132130
0 2 4 6 7	CDEF#G	2 2 2 1 (5)	incomplete	open	5-24*	131221
0 2 4 6 8	CDEF#G#	2 2 2 2 (4)	incomplete	open	5-33	040402
0 1 3 4 6 7	CDbEbEF#G#	1 2 1 2 1 (5)	incomplete	open	6-Z13	324222
0 1 3 4 6 8	CDbEbEF#G#	1 2 1 2 2 (4)	incomplete	open	6-Z24	233331
0 1 3 5 6 8	CDbEbFGbAb	1 2 2 1 2 (4)	incomplete	open	6-Z25	233241
0 1 3 5 7 8	CDbEbFGAb	1 2 2 2 1 (4)	incomplete	open	6-Z26	232341
0 1 3 5 7 9	CDbEbFGA	1 2 2 2 2 (3)	incomplete	open	6-34	142422
0 2 3 5 6 8	CDEbFGbAb	2 1 2 1 2 (4)	incomplete	open	6-Z23	234222
0 2 3 5 7 8	CDEbFGAb	2 1 2 2 1 (4)	incomplete	open	6-Z25*	233241
0 2 3 5 7 9	CDEbFGA	2 1 2 2 2 (3)	incomplete	open	6-33	143241
0 2 4 5 7 8	CDEFGAb	2 2 1 2 1 (4)	incomplete	open	6-Z24*	233331
0 2 4 5 7 9	CDEFGA	2 2 1 2 2 (3)	incomplete	open	6-32	143250
0 2 4 6 7 9	CDEF#GA	2 2 2 1 2 (3)	incomplete	open	6-33*	143241
0 2 4 6 8 9	CDEF#G#A	2 2 2 2 1 (3)	incomplete	open	6-34*	142422
0 1 3 4 6 7 9	CDbEbEF#GA	1 2 1 2 1 2 (3)	incomplete	open	7-31	336333
0 1 3 4 6 8 9	CDbEbEF#G#A	1 2 1 2 2 1 (3)	complete	open	7-32	335442
0 1 3 5 6 8 9	CDEbFGbAbA	1 2 2 1 2 1 (3)	complete	open	7-32*	335442
0 2 3 5 6 8 9	CDEbFF#G#A	2 1 2 1 2 1 (3)	incomplete	open	7-31*	336333

**Example 3: A list of the proper closed step-collection classes (rotations listed together).**

<i>pitch integers</i>	<i>letter names</i>	<i>adjacent intervals</i>	<i>complete?</i>	<i>closed?</i>	<i>Forte name</i>	<i>vector</i>
0 2 4 6 8 10	CDEF#G#A#	2 2 2 2 2 (2)	complete	closed	6-35	060603
0 1 3 4 6 8 10	CDbEbFbGbAbBb	1 2 1 2 2 2 (2)	complete	closed	7-34	254442
0 1 3 5 7 9 10	CDbEbFGABb	1 2 2 2 2 1 (2)	complete	closed	7-34+	254442
0 2 3 5 6 8 10	CDEbFGbAbBb	2 1 2 1 2 2 (2)	complete	closed	7-34+	254442
0 2 3 5 7 9 11	CDEbFGAB	2 1 2 2 2 2 (1)	complete	closed	7-34+	254442
0 2 4 5 7 8 10	CDEFGAbBb	2 2 1 2 1 2 (2)	complete	closed	7-34+	254442
0 2 4 6 7 9 10	CDEF#GABb	2 2 2 1 2 1 (2)	complete	closed	7-34+	254442
0 2 4 6 8 9 11	CDEF#G#AB	2 2 2 2 1 2 (1)	complete	closed	7-34+	254442
0 1 3 5 6 8 10	CDbEbFGbAbBb	1 2 2 1 2 2 (2)	complete	closed	7-35	254361
0 1 3 5 7 8 10	CDbEbFGAbBb	1 2 2 2 1 2 (2)	complete	closed	7-35+	254361
0 2 3 5 7 8 10	CDEbFGAbBb	2 1 2 2 1 2 (2)	complete	closed	7-35+	254361
0 2 3 5 7 9 10	CDEbFGABb	2 1 2 2 2 1 (2)	complete	closed	7-35+	254361
0 2 4 5 7 9 10	CDEFGABb	2 2 1 2 2 1 (2)	complete	closed	7-35+	254361
0 2 4 5 7 9 11	CDEFGAB	2 2 1 2 2 2 (1)	complete	closed	7-35+	254361
0 2 4 6 7 9 11	CDEF#GAB	2 2 2 1 2 2 (1)	complete	closed	7-35+	254361
0 1 3 4 6 7 9 10	CDbEbEF#GABb	1 2 1 2 1 2 1 (2)	complete	closed	8-28	448444
0 2 3 5 6 8 9 11	CDEbFF#G#AB	2 1 2 1 2 1 2 (1)	complete	closed	8-28	448444

All step collections of less than six notes are incomplete and open. The six-note whole-tone scale is complete and closed, but all other six-note collections are incomplete and open. Two proper seven-note step collections (represented by the diatonic collection and the “ascending melodic minor” scale) are complete and closed, two (the two seven-note subsets of the octatonic scale) are incomplete and open, and two (represented by the “harmonic minor” scale, and what might be called the “harmonic major” scale) are complete but open. The only proper eight-note step collection is the octatonic scale, which is complete and closed. The fourth column of Examples 2 and 3 indicates whether the step collection is complete or incomplete. The fifth column of Examples 2 and 3 indicates whether the step collection is open or closed.

The sixth column of Examples 2 and 3 lists the name given by Forte [1973, 179-181] to the entry in his Appendix 1 that matches the first column of Examples 2 and 3. Forte’s table is a list of “pitch-class sets” (what Rahn [1980] calls “ $T_n/T_nI$  set types” and Mead [1983] calls “collection classes”). Because Forte’s labels describe sets of *pitch classes* that are *equivalent* under inversion—and the step collections are sets of *pitches* that are *not equivalent* under inversion—more than one step-collection class can share one of Forte’s labels. Where the first column of Example 2 or 3 matches the second column of Forte’s table, the sixth column gives Forte’s label. Where the first column of Example 2 or 3 gives a collection whose inversion would match the second column of Forte’s table, the sixth column gives Forte’s label followed by an asterisk. Where the first column of Example 2 or 3 gives a collection whose rotation would match the second column of Forte’s table, the plus sign indicates that a rotation of the step-collection class would match the second column of Forte’s table.

### Leap Collections

A "leap collection" is another special set of pitches. In a leap collection, no two pitches are a step apart. In a proper leap collection, no two pitches nor any of their octave equivalents are a step apart. The leap collections may be placed in "leap-collection classes"—classes of equivalence under transposition (again, for reasons that will become clear, these classes, like those for the step collections, are not equivalent under inversion). Example 4 lists all proper leap-collection classes.

If no pitch can be added to a leap collection to produce another leap collection, then it is called "complete". If a pitch can be added to a leap collection to produce another leap collection, then it is called "incomplete". The distinction between "open and closed" step collections does not apply to leap collections.

#### Example 4: A list of the proper leap-collection classes.

<i>pitches</i>	<i>letter names</i>	<i>adjacent intervals</i>	<i>complete?</i>	<i>Forte name</i>	<i>vector</i>
0 3	CEb	3 (9)	incomplete		001000
0 4	CE	4 (8)	incomplete		000100
0 5	CF	5 (7)	incomplete		000010
0 6	CF#	6 (6)	incomplete		000001
0 7	CG	7 (5)	incomplete	+	000010
0 8	CAb	8 (4)	incomplete	+	000100
0 9	CA	9 (3)	incomplete	+	001000
0 3 6	CEbF#	3 3 (6)	incomplete	3-10	002001
0 3 7	CEbG	3 4 (5)	complete	3-11	001110
0 3 8	CEbAb	3 5 (4)	complete	3-11*+	001110
0 3 9	CEbA	3 6 (3)	incomplete	3-10+	002001
0 4 7	CEG	4 3 (5)	complete	3-11*	001110
0 4 8	CEAb	4 4 (4)	complete	3-12	000300
0 4 9	CEA	4 5 (3)	complete	3-11+	001110
0 5 8	CFAb	5 3 (4)	complete	3-11+	001110
0 5 9	CFA	5 4 (3)	complete	3-11*+	001110
0 6 9	CF#A	6 3 (3)	incomplete	3-10+	002001
0 3 6 9	CEbF#A	3 3 3 (3)	complete	4-28	004002

### Frames

A stable leap collection may serve as a "frame" (or "tonality frame"). A frame provides the stable reference points for a melody. Some textbooks for written and aural skills explicitly describe and exploit frames [Christ, et al 1973; Larson; Thomson 1969; Thomson 1970; and Thomson 1980]. Others, especially those that draw on principles of Schenker's theories, use the idea implicitly [e.g., Aldwell and Schachter 1989; Gould 1979]. Arnheim has called analogous structures in visual arts "structural skeletons" [1974]. Melodies generate internal representations of frames, and frames give expression to melodies. The dynamic tendencies in a frame emerge most clearly when stepwise patterns connect notes of that frame.

### *Musical forces*

When we listen to tonal music, we experience certain “forces”. These forces include “musical gravity”, “musical magnetism”, “musical inertia”. The operation of these is governed by a principle of “pattern completion”. Together, they are responsible for some of the dynamic tendencies we experience in listening to music. Each may be explained in terms of the principles of perceptual organization articulated by Gestalt psychologists. Together, they afford necessary and sufficient conditions for the emergence of scale-degree function.

### *Musical gravity*

We hear musical motion taking place in “musical space”. The metaphor of musical space is a natural one. A variety of spatial models have been proposed in the music-cognition literature [Balzano 1980; Deutsch 1969; Krumhansl 1990; Lerdahl 1988; Longuet-Higgins 1978; Shepard 1982], but the metaphor is clear in earlier writing, too (as just one example, consider Schenker’s idea of *Tonraum*). Musical space is asymmetrical—it has an “up and down”. Arnheim [1986] discusses this “anisotropy of musical space” and argues that we experience dynamic tendencies in both visual art and music that parallel the tendencies of gravity in physical space. “Musical gravity” is thus the tendency of an unstable note to descend to a more stable note. Whether it is learned or innate, our perception of musical gravity is immediate, and it is central to expression in tonal music. It is because of this asymmetry of musical space that the step-collection classes and the leap-collection classes are not equivalent under inversion. My own informal tests suggest that equivalence under transposition is not perceptually robust. Published studies raise even larger questions about the perceptual significance of such equivalence classes [Gibson 1986 and 1988].

### *Musical magnetism*

“Musical magnetism” is the tendency of an unstable note to move (up or down) to the nearest stable pitch. Magnetism is affected by distance—the closer we get to a goal, the more it attracts us. Magnetism appears to influence intonation and intonation judgements (Frances 1958/1988). Analogous forces operate in visual art [Arnheim 1974 and 1986].

### *Musical inertia*

“Musical inertia” is the tendency of a pattern of musical motion to continue in the same direction. Gestalt psychology calls this tendency “the law of good continuation” [Koehler 1947]. Music psychologists have drawn on Gestalt perceptual principles to explain grouping and melody [e.g., Bregman 1976; Deutsch 1982; Lerdahl 1983; Sloboda 1985—see Butler 1992]. Meyer’s work on emotion in music [1956] draws upon the confirmation or denial of expected continuations. Browne extends the principle of “good continuation”.

### *Pattern completion*

Each of the above-mentioned forces may be understood as suggesting that the dynamic tendencies of a note are shaped by its participation in patterns. Another Gestalt tendency is the drive toward closure. Anytime we can imagine a more complete or more stable pattern than the one we are hearing, we hear what we perceive in terms of that pattern and experience a desire to hear it completed or made more stable.

### *Scale-degree function*

Each of the musical forces acts on notes at all times. The less stable a note, the more that note will convey to the listener the effects of these forces. The distinctive combination of these effects in a given context contributes to the expressive quality of that note in its context. This combination of felt tendencies also contributes to what we call "scale-degree function".

"Scale-degree function" is central to music cognition and music pedagogy. This scale-degree function depends on the perceived stability of its note and may be described as a mixture of audiated resolution tendencies. Example 5 illustrates only some of these tendencies. Students who can identify scale-degree function can more easily be taught basic aural skills. The best way to build skill at identifying scale-degree function is to learn the tendencies of scale degrees in prototypical situations. The three- and four-note patterns described below claim to be the best instances of such prototypical situations.

**Example 5: Some tendencies of diatonic scale degrees in the context of tonic harmony.**

Ex 5

^7      ^1      ^2      ^3      ^4      ^5      ^6

### *Three- and four-note patterns*

The operation of the musical forces described above leads to the generation of the set of three- and four-note patterns shown in Examples 6 and 7. These patterns meet the following conditions (where "stable note" means a member of a tonic-triad frame): (1) each pattern begins on a stable note, (2) each pattern moves by step, and (3) each pattern ends on a stable note. (These patterns also meet an additional condition—that none contains both  $\wedge 3$  and  $\wedge b3$ . The fact that this condition seems less "necessary" in terms of the assumptions outlined above may explain why it seems one of the first conditions to be relaxed in less classically tonal music of the late nineteenth and early twentieth centuries, which makes use of patterns that not only meet the three numbered conditions, but that also mix  $\wedge 3$  and  $\wedge b3$ .)



Example 6 catalogues the three-note patterns. The first column describes the pattern as it would be understood in the context of its tonic-triad frame. The second column gives letter-name equivalents in the key of C. The third column identifies the step collection created by the pattern, considered alone (this column may be cross-referenced with the first column of Example 2). The fourth column indicates which forces predict the actual third note. (An example may make this clearer. In the pattern  $\wedge 5-\wedge\#4-\wedge 3$ , the inertia developed by the first two notes tends to continue the line in the same direction, to  $\wedge 3$ ; thus, inertia predicts the third note. In the same pattern, gravity pulls the unstable  $\wedge\#4$  down, to  $\wedge 3$ ; thus, gravity also predicts the third note. And in the same pattern, magnetism pulls the unstable  $\wedge\#4$  to the nearest stable note, to  $\wedge 5$ ; thus, magnetism does not predict the third note of the pattern  $\wedge 5-\wedge\#4-\wedge 3$ .) Where the final motion is a half step, the magnetic pull is strongest; this is indicated by an exclamation point after the word "magnetism". When stable notes can be found both a whole step above and a whole step below a pattern, we may experience motion to either note as a giving in to a magnetic pull; this is indicated by a question mark after the word "magnetism".

**Example 6: The three-note patterns.**

<i>scale degrees</i>	<i>letter names in C</i>	<i>step collections</i>	<i>forces which predict the third note</i>		
$\wedge 5-\wedge 6-\wedge 5$	GAG	0 2	gravity	magnetism	
$\wedge 5-\wedge 6-\wedge 5$	GAbG	0 1	gravity	magnetism!	
$\wedge 5-\wedge\#4-\wedge 5$	GF#G	0 1		magnetism!	
$\wedge 5-\wedge\#4-\wedge 3$	GF#E	0 2 3	gravity		inertia
$\wedge 5-\wedge 4-\wedge 5$	GFG	0 2			
$\wedge 5-\wedge 4-\wedge 3$	GFE	0 1 3	gravity	magnetism!	inertia
$\wedge 5-\wedge 4-\wedge 6b$	GFEB	0 2 4	gravity	magnetism	inertia
$\wedge 3-\wedge\#4-\wedge 5$	EF#G	0 2 3		magnetism	inertia
$\wedge 3-\wedge\#4-\wedge 3$	EF#E	0 2			
$\wedge 3-\wedge 4-\wedge 5$	EFG	0 1 3			inertia
$\wedge 3-\wedge 4-\wedge 3$	EFE	0 1	gravity	magnetism!	
$\wedge 3-\wedge\#2-\wedge 3$	ED#E	0 1		magnetism!	
$\wedge 3-\wedge 2-\wedge 3$	EDE	0 2		magnetism?	
$\wedge 3-\wedge 2-\wedge 1$	EDC	0 2 4	gravity	magnetism?	inertia
$\wedge 6b-\wedge 4-\wedge 5$	EbFG	0 2 4		magnetism?	inertia
$\wedge 6b-\wedge 4-\wedge 6b$	EbFEb	0 2	gravity	magnetism?	
$\wedge 6b-\wedge 2-\wedge 6b$	EbDEb	0 1		magnetism!	
$\wedge 6b-\wedge 2-\wedge 1$	EbDC	0 2 3	gravity		inertia
$\wedge 6b-\wedge 6b-\wedge 6b$	EbDbEb	0 2			
$\wedge 6b-\wedge 6b-\wedge 1$	EbDbC	0 1 3	gravity	magnetism!	inertia
$\wedge 1-\wedge 2-\wedge 3$	CDE	0 2 4		magnetism?	inertia
$\wedge 1-\wedge 2-\wedge 6b$	CDEb	0 2 3		magnetism!	inertia
$\wedge 1-\wedge 2-\wedge 1$	CDC	0 2	gravity	magnetism?	
$\wedge 1-\wedge 6b-\wedge 6b$	CDbEb	0 1 3			inertia
$\wedge 1-\wedge 6b-\wedge 1$	CDbC	0 1	gravity	magnetism!	
$\wedge 8-\wedge 7-\wedge 8$	CBC	0 1		magnetism!	
$\wedge 8-\wedge 7-\wedge 8$	CBbC	0 2		magnetism	

Example 7 catalogues the four-note patterns. The patterns that end with the half step (the upper tetrachords of the ascending and descending melodic minor scale, marked with an exclamation point after the word "magnetism") are felt as most strongly directed toward their goal.

**Example 7: The four-note patterns.**

<i>scale degrees</i>	<i>letter names in C</i>	<i>step collections</i>	<i>forces which predict the fourth note</i>		
$\wedge 5 \wedge 6 \wedge 7 \wedge 8$	GABC	0 2 4 5		magnetism!	inertia
$\wedge 5 \wedge 6 \wedge 7 \wedge 8$	GABbC	0 2 3 5		magnetism	inertia
$\wedge 5 \wedge 6 \wedge 7 \wedge 8$	GAbBbC	0 1 3 5		magnetism	inertia
$\wedge 8 \wedge 7 \wedge 6 \wedge 5$	CBAG	0 2 4 5	gravity	magnetism	inertia
$\wedge 8 \wedge 7 \wedge 6 \wedge 5$	CBbAG	0 2 3 5	gravity	magnetism	inertia
$\wedge 8 \wedge 7 \wedge 6 \wedge 5$	CBbAbG	0 1 3 5	gravity	magnetism!	inertia

The table in Example 8 categorizes the three-note patterns by shape (columns) and starting note (rows).

Since goal-direction is an important aspect of tonal music, we should expect that the patterns in which the final note is most strongly predicted by the musical forces are the patterns that occur most frequently in tonal music. As an example, consider the pattern  $\wedge 5 \wedge 4 \wedge 5$ . This pattern defies gravity, magnetism, and inertia. And this pattern is unusual in tonal melody. When  $\wedge 5$  has a lower neighbor, it is usually  $\wedge 5 \wedge \#4 \wedge 5$  (so that magnetism may be heard to overcome gravity and inertia). While  $\wedge 4$  may go to  $\wedge 5$  in the pattern  $\wedge 3 \wedge 4 \wedge 5$  (so that inertia may be heard to overcome gravity and magnetism), when  $\wedge 5$  moves to  $\wedge 4$  it usually continues with gravity, magnetism, and inertia as the pattern  $\wedge 5 \wedge 4 \wedge 3$ .

The patterns in which the final note is predicted most strongly by these forces are also the patterns in which the unstable notes have their most characteristic tendencies—in which their clearest scale-degree function emerges. A ranking of the patterns thus emerges. For example, Example 9 (on page 12) first groups each three-note pattern under the unstable scale degree that it includes (in two somewhat artificial columns—one for major and one for minor). Within each such group, the patterns are listed in the order of their "strength". Here, strength means the degree to which the final note is predicted by gravity, magnetism, and inertia.

**Example 8: The three-note patterns, classified by shape (columns) and starting note (rows).**

<i>UU (asc passing tone)</i>	<i>UD (upper neighbor)</i>	<i>DU (lower neighbor)</i>	<i>DD (desc passing tone)</i>
	$\wedge 5-\wedge 6-\wedge 5$	$\wedge 5-\wedge \#4-\wedge 5$	$\wedge 5-\wedge \#4-\wedge 3$
	$\wedge 5-\wedge b6-\wedge 5$	$\wedge 5-\wedge 4-\wedge 5$	$\wedge 5-\wedge 4-\wedge 3$
			$\wedge 5-\wedge 4-\wedge b3$
$\wedge 3-\wedge \#4-\wedge 5$	$\wedge 3-\wedge \#4-\wedge 3$	$\wedge 3-\wedge \#2-\wedge 3$	$\wedge 3-\wedge 2-\wedge 1$
$\wedge 3-\wedge 4-\wedge 5$	$\wedge 3-\wedge 4-\wedge 3$	$\wedge 3-\wedge 2-\wedge 3$	
$\wedge b3-\wedge 4-\wedge 5$	$\wedge b3-\wedge 4-\wedge b3$	$\wedge b3-\wedge 2-\wedge b3$	$\wedge b3-\wedge 2-\wedge 1$
		$\wedge b3-\wedge b2-\wedge b3$	$\wedge b3-\wedge b2-\wedge 1$
$\wedge 1-\wedge 2-\wedge 3$	$\wedge 1-\wedge 2-\wedge 1$	$\wedge 8-\wedge 7-\wedge 8$	
$\wedge 1-\wedge 2-\wedge b3$	$\wedge 1-\wedge b2-\wedge 1$	$\wedge 8-\wedge b7-\wedge 8$	

**Example 9: A ranking of three-note patterns.**

<i>unstable note</i>	<i>major</i>	<i>forces which predict the third note</i>			<i>minor</i>
$\wedge 6$	$\wedge 5 \wedge 6 \wedge 5$	gravity	magnetism		
$\wedge b 6$		gravity	magnetism!		$\wedge 5 \wedge b 6 \wedge 5$
$\wedge \# 4$	$\wedge 3 \wedge \# 4 \wedge 5$ $\wedge 5 \wedge \# 4 \wedge 5$ $\wedge 5 \wedge \# 4 \wedge 3$ $\wedge 3 \wedge \# 4 \wedge 3$	gravity gravity	magnetism! magnetism!	inertia inertia	$\wedge 5 \wedge \# 4 \wedge 5$
$\wedge 4$	$\wedge 5 \wedge 4 \wedge 3$ $\wedge 3 \wedge 4 \wedge 3$ $\wedge 3 \wedge 4 \wedge 5$ $\wedge 5 \wedge 4 \wedge 5$	gravity gravity gravity gravity	magnetism! magnetism? magnetism! magnetism?	inertia inertia inertia inertia	$\wedge 5 \wedge 4 \wedge b 3$ $\wedge b 3 \wedge 4 \wedge b 3$ $\wedge b 3 \wedge 4 \wedge 5$ $\wedge 5 \wedge 4 \wedge 5$
$\wedge \# 2$	$\wedge 3 \wedge \# 2 \wedge 3$		magnetism!		
$\wedge 2$	$\wedge 3 \wedge 2 \wedge 1$ $\wedge 1 \wedge 2 \wedge 3$ $\wedge 1 \wedge 2 \wedge 1$ $\wedge 3 \wedge 2 \wedge 3$	gravity gravity gravity gravity	magnetism? magnetism! magnetism? magnetism! magnetism?	inertia inertia inertia inertia	$\wedge b 3 \wedge 2 \wedge 1$ $\wedge 1 \wedge 2 \wedge b 3$ $\wedge 1 \wedge 2 \wedge 1$ $\wedge b 3 \wedge 2 \wedge b 3$
$\wedge b 2$		gravity gravity	magnetism! magnetism!	inertia inertia	$\wedge b 3 \wedge b 2 \wedge 1$ $\wedge 1 \wedge b 2 \wedge 1$ $\wedge 1 \wedge b 2 \wedge b 3$ $\wedge b 3 \wedge b 2 \wedge b 3$
$\wedge 7$	$\wedge 8 \wedge 7 \wedge 8$		magnetism!		$\wedge 8 \wedge 7 \wedge 8$
$\wedge b 7$			magnetism		$\wedge 8 \wedge b 7 \wedge 8$

### Combining patterns

These three- and four-note patterns may be combined successively or simultaneously.

The successive combination of these patterns forms an important basis of tonal melody. If the last note of a pattern is the same as the first note of a following pattern, their combination may elide that note (for example, joining two three-note patterns to form a five-note pattern). Example 10 illustrates the combination of patterns. Example 10a gives the three-note pattern  $\wedge^3-\wedge^4-\wedge^3$ . Example 10b gives the three-note pattern  $\wedge^3-\wedge^2-\wedge^1$ . Example 10c gives their five-note combination  $\wedge^3-\wedge^4-\wedge^3-\wedge^2-\wedge^1$ . Such elided patterns create smooth, directed melodic motion. When two three-note patterns are combined to form a five-note pattern this way, if the inertia of the second to third note (the last two notes of the first pattern) predicts the direction of the third to fourth notes (the first two notes of the second pattern), the result is even smoother. Example 11 shows a string of elided patterns, each introduced by the inertia of the preceding pattern. The string in Example 11 is canonical in two senses: (1) the melodic patterns that occur most frequently at all levels of melodic structure are embellishments of portions of this string, and (2) the string may be sung as a canon so that adjacent patterns are sung simultaneously.

Example 10: An illustration of the successive combination of patterns.

Ex 10

The musical notation for Example 10 consists of a single staff with a treble clef. It is divided into three measures by vertical bar lines. Above the staff, scale degree symbols are placed above each note. Measure 'a' contains three notes:  $\wedge^3$ ,  $\wedge^4$ , and  $\wedge^3$ . Measure 'b' contains three notes:  $\wedge^3$ ,  $\wedge^2$ , and  $\wedge^1$ . Measure 'c' contains five notes:  $\wedge^3$ ,  $\wedge^4$ ,  $\wedge^3$ ,  $\wedge^2$ , and  $\wedge^1$ . The notes in each measure are connected by curved lines (slurs) indicating melodic flow.

Example 11: Elided patterns, each introduced by the inertia of the preceding pattern.

Ex 11

The musical notation for Example 11 consists of a single staff with a treble clef. Above the staff, scale degree symbols are placed above each note. The sequence of notes is:  $\wedge^1$ ,  $\wedge^7$ ,  $\wedge^1$ ,  $\wedge^2$ ,  $\wedge^3$ ,  $\wedge^4$ ,  $\wedge^5$ ,  $\wedge^6$ ,  $\wedge^5$ ,  $\wedge^4$ ,  $\wedge^3$ ,  $\wedge^2$ ,  $\wedge^3$ ,  $\wedge^4$ ,  $\wedge^3$ ,  $\wedge^2$ ,  $\wedge^1$ ,  $\wedge^7$ , and  $\wedge^8$ . The notes are connected by a series of overlapping curved lines (slurs) that create a continuous, flowing melodic line.

The simultaneous combination of patterns forms an important basis of tonic prolongations. Example 12 shows combinations that appear frequently in the outer voices of tonic prolongations in two or more voices. (The appearance here of two patterns that don't appear in Example 7—the incomplete neighbor patterns  $\hat{1}-\hat{4}-\hat{3}$  and  $\hat{3}-\hat{7}-\hat{8}$ —may suggest shortcomings in or possible improvements to the model.)

Several fruitful combination of simultaneous patterns, which I call “solfege drills”, are given in Example 13. Each of these combinations meets the following criteria: (1) their patterns collectively produce a step-collection, (2) each unstable note in the combination appears in one and only one of the patterns, (3) the patterns form a prototypical harmonic progression (noted beneath the pattern).

### *Embellishing the patterns*

These patterns (and their combinations) provide shapes that are simple and directed. This directed quality is a source of what we call scale-degree function. The organizing power of these simple shapes also makes them ideal sources for improvisation. They are easy to embellish because they provide a secure path that is easy to hear and return to. And they teach embellishment by example because they themselves are simple embellishments.

### *Patterns in tonal music*

These specific two- and three-note patterns enjoy a privileged status in the structure of tonal music. Tonal melody relies heavily on the combination and embellishment of these patterns.

### *Recommendations for teaching aural skills*

This model of music cognition suggests that the construction, identification, combination, and embellishment of these patterns should be a central part of the training of musicians. Example 14 lists several recommendations for teaching aural skills.

**Example 12: Combinations that appear frequently in the outer voices of tonic prolongations.**

		SOPRANO		
		<i>desc passing tone</i>	<i>asc passing tone</i>	<i>inc neighbor</i>
BASS	<i>desc passing tone</i>	$\wedge 5-\wedge 4-\wedge 3$ $\wedge 3-\wedge 2-\wedge 1$	$\wedge 1-\wedge 2-\wedge 3$ $\wedge 3-\wedge 2-\wedge 1$	$\wedge 1-\wedge 4-\wedge 3$ $\wedge 3-\wedge 2-\wedge 1$
		10-10-10 parallel tenths	6-8-10 voice exchange	6-10-10 voice exchange
	<i>asc passing tone</i>	$\wedge 3-\wedge 2-\wedge 1$ $\wedge 1-\wedge 2-\wedge 3$	$\wedge 3-\wedge 4-\wedge 5$ $\wedge 1-\wedge 2-\wedge 3$	$\wedge 3-\wedge 7-\wedge 8$ $\wedge 1-\wedge 2-\wedge 3$
		10-8-6 voice exchange	10-10-10 parallel tenths	10-6-6 voice exchange
	<i>inc neighbor</i>	$\wedge 5-\wedge 4-\wedge 3$ $\wedge 3-\wedge 7-\wedge 8$	$\wedge 1-\wedge 2-\wedge 3$ $\wedge 3-\wedge 7-\wedge 8$	$\wedge 1-\wedge 4-\wedge 3$ $\wedge 3-\wedge 7-\wedge 8$
		10-*5-10	6-10-10 voice exchange	6-*5-10 voice exchange
		$\wedge 3-\wedge 2-\wedge 1$ $\wedge 1-\wedge 4-\wedge 3$		$\wedge 3-\wedge 7-\wedge 8$ $\wedge 1-\wedge 4-\wedge 3$
		10-6-6 voice exchange		10-*4-6 voice exchange

**Example 13: Combinations of three-note patterns—Solfege drills.***three-note patterns in major*

^5	^4	^3
^3	^2	^1
^8	^7	^8
I	V7	I

*three-note patterns in minor*

^5	^4	^b3
^b3	^2	^1
^8	^7	^8
i	V7	i

*three- and four-note patterns in major*

^5	^6	^7	^8
^5	^6	^5	^1
^5	^4	^4	^3
^3	^2	^2	^1
^8	^8	^7	^8
I	ii6 5	V7	I

*three- and four-note patterns in minor*

^5	^b6	^5	^1
^5	^4	^4	^b3
^b3	^2	^2	^1
^8	^8	^7	^8
i	ii <sup>o</sup> 6 5	V7	i



**Example 14: Two-dozen suggestions for the pedagogy of musicianship skills.**

1. Given one note of a step collection and a description of the step collection, sing the rest of the step collection. You may wish to work on all the step collections of a given cardinality before working on others, or work on the step collections that are subsets of a single, larger collection (such as the diatonic collection), before working on others.
2. Pick a step collection. Call one of the notes "do" or " $\wedge 1$ ". Sing the rest of the collection with solfege syllables.
3. Have a friend perform a step collection in list order. Identify the step collection.
4. Have a friend perform a step collection in varied order. Sing the step collection in list order. Identify the step collection. For complete step collections, repeat this exercise with their rotations.
5. Given one note of a leap collection and a description of the leap collection, sing the rest of the leap collection.
6. Pick a leap collection. Call one of the notes "do" or " $\wedge 1$ ". Sing the rest of the collection with solfege syllables.
7. Have a friend perform a leap collection in list order. Identify the leap collection.
8. Have a friend perform a leap collection in varied order. Sing the leap collection in list order. Identify the leap collection.
9. Play a perfect fifth on the piano with your left hand. Above this  $\wedge 5/\wedge 1$  frame, improvise a brief piece with a clear beginning, middle, and end. Be sure to do this exercise in various keys. (No additional explicit constraints.)
10. Play another improvisation above a  $\wedge 5/\wedge 1$  frame, but with one or more of the following constraints:
  - (a) Sing each note as you play it.
  - (b) Sing each gesture after you play it.
  - (d) Sing each gesture before you play it.
  - (e) Sing each gesture with solfege after you play it.
  - (f) Sing each gesture with solfege before you play it.
  - (g) Sing each gesture with solfege as you play it.
  - (h) Limit the pitches to one specified step collection.
  - (i) Use one or more three- or four-note patterns as the basis of all or part of your improvisation.
11. Given the first two notes of a pattern and a musical force, sing and then write the notes predicted by that force. (See Example 14.)
12. Play a perfect fifth on the piano with your left hand. Above this  $\wedge 5/\wedge 1$  frame, sing each of the three- and four-note patterns given in Examples 6 and 7. Looking only at the first column, determine which musical forces predict the final note. Check your answer against the fourth column. As you sing each pattern, feel the effects of the combined forces—perform the pattern in a way that reflects the changing tension in it. Repeat, omitting the first note of each pattern.
13. Have a friend establish a tonal center (possibly by playing a  $\wedge 5/\wedge 1$  frame) and then perform one or more patterns. Sing back the pattern. Sing back the pattern with solfege. Echo the pattern on the piano or on your

instrument. Identify the pattern by pointing to Example 6, 7, or 8.

14. Make up a game involving the elision of patterns. (One of the rules might concern how the direction of the last two notes of one pattern influences the direction of the first two notes of the following pattern.) Play your invented game.

15. Practice singing and playing the combined patterns in Example 12. Let your performance of these patterns reflect the dynamic tendencies created by the musical forces that shape it.

16. Have a friend perform one of the pattern combinations in Example 12. Sing back the separate voices of the pattern-combination. Sing back the separate voices with solfege. Echo the pattern-combination on the piano or on your instrument. Identify the pattern by pointing to Example 12.

17. Have a friend perform one of the soprano voices from Example 12 twice. On the second performance, add one of the bass voices that appears with that soprano in Example 12. Point in Example 12 to the pattern combination you created.

18. Make up a game that is a variation on Exercise 17.

19. While playing the bass line indicated by the Roman numerals in Example 13, sing one of the upper voices with solfege syllables. Let your performance of these patterns reflect the dynamic tendencies created by the musical forces that shape it.

20. Before and/or after sightsinging or practicing a passage of music:  
(a) Identify step collections and leap collections used.  
(b) Identify the frame or frames of each of the parts.  
(c) Find three- and four-note patterns and their embellishments.  
(d) Suggest ways in which a performance could reflect the dynamic tendencies created by the musical forces that shape it.

21. With a group of friends, sing each of the patterns in one of the combinations given in Example 13 in turn. Begin at different times to create a round.

22. Alternate performances of patterns as in Exercise 21 with improvised embellishments of the same patterns (preserving the underlying rhythm).

23. Have each member of a group of friends sing a different pattern contained in one of the combinations given in Example 13. Repeat the pattern rhythmically. Each group member may embellish some or all of the repetitions of his or her pattern. Or begin at different times to create an embellished round.

24. Sing one of the patterns from a combination given in Examples 12 or 13 while playing another.

**Example 15: A sample exercise (Exercise 11 from Example 14).**

Patterns starting on  $\overset{\wedge}{2}$

Exc 11

2 3 4 5 6

gravity magnetism inertia gravity magnetism inertia

7 8 9 10 11 12

gravity magnetism inertia gravity magnetism inertia

Patterns starting on  $\overset{\wedge}{3}$

13 14 15 16 17 18

gravity magnetism inertia gravity magnetism inertia

19 20 21 22 23 24

gravity magnetism inertia gravity magnetism inertia

Patterns starting on  $\overset{\wedge}{b}3$

25 26 27 28 29 30

gravity magnetism inertia gravity magnetism inertia

31 32 33

gravity magnetism inertia

Patterns starting on  $\overset{\wedge}{1}$

34 35 36 37 38 39

gravity magnetism inertia gravity magnetism inertia

40 41 42 43 44 45

gravity magnetism inertia gravity magnetism inertia

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