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A Network Analysis of Children's Emerging Place-Value Concepts

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Abstract

Examining how informal knowledge systems change after formal instruction is imperative to understanding learning processes and conceptual development and to implementing effective educational practices. We used network analyses to determine how the organization of informal knowledge about multidigit numbers in kindergartners ($N = 279$; mean age = 5.76 years, $SD = 0.55$; 135 females) supports and is transformed by a year of in-school formal instruction. The results show that in kindergarten, piecemeal knowledge about the surface properties of reading and writing multidigit numbers and the use of base-10 units to determine large quantities are strongly associated with each other and connected in a stringlike manner to other emerging skills. After a year of instruction, each skill becomes connected to the "hub" abilities of reading and writing multidigit numbers, which also become strongly connected to more advanced knowledge of base-10 principles. These findings provide new insights into how partial knowledge provides the backbone on which explicit principles are learned.

Keywords

knowledge structure, place-value understanding, multidigit numbers, mathematical learning, network analysis, open data

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Knowledge in mathematics and science is built incrementally, beginning piecemeal but becoming an integrated system capable of generating new insights. Unschooled concepts have been characterized both as foundational (Carey, 2001) and as impediments (McCloskey, 2014) to advanced learning. Here, we examined how early approximate and rudimentary knowledge may support and be transformed by formal instruction. Theories of knowledge change often use network analyses to characterize and measure how components are integrated into a single system (see Baronchelli et al., 2013; Siew, 2020). Here, we used network analyses to understand how children's early piecemeal knowledge transforms into a more integrated understanding of place value.

Place value is a compositional system consisting of a small set of symbols (the digits 0 to 9) and syntactic principles that generate and relate representations for an infinite set of quantities. The syntax expresses base-10 relations in terms of units arranged hierarchically in multiples of 10 from right to left, with counts of each unit represented by the specific digit in each place. For example, "532" stands for [5 × 100] + [3 × 10] + [2 × 1]. This *count + unit syntax* is difficult for many, but not all, children to master (e.g., Chan et al., 2014; Fuson, 1990), and incomplete understanding is associated with persistent difficulties in syntax-based operations such as multidigit calculation (Moeller et al., 2011; Raghubar et al., 2009).

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Children begin forming ideas about multidigit numbers, and specifically about how number names map to written forms, well before the start of school (Mix et al., 2014; Yuan et al., 2019). At levels above chance, children as young as 3 and 4 years of age map number names to written forms, correctly judging, for example, that "five hundred thirty-two" maps onto "532" instead of "325" (Mix et al., 2014; Yuan et al., 2019). They sometimes make errors and are more likely to do so when target and foils are transpositions or differ only in the presence of a zero—errors that show that their knowledge is at best incomplete (Yuan et al., 2020). Many preschool children also correctly write the multidigit number in a dictation task when given the spoken number name (Byrge et al., 2014). Preschool children also make interesting errors in this task that indicate a not-quite-right understanding. For example, they may write "six hundred and forty-two" as "600402" (Byrge et al., 2014; Zuber et al., 2009). Growing evidence indicates that early knowledge about number names and their corresponding written forms emerges from incidental experiences with the names and forms (Byrge et al., 2014; Mix et al., 2014; Yuan et al., 2019, 2020). Reading and writing multidigit numbers, even imperfectly, would seem a useful first step for learning about place value. However, these *transcoding* skills, as they are often referred to in the mathematics-cognition literature (e.g., Zuber et al., 2009), can be accomplished with no knowledge of count + unit syntax (Yuan et al., 2019, 2020). All English speakers need to know is that the names of the digits and the temporal order of the spoken names (with the exceptions of the teens) align, from first mentioned to last mentioned, with the left-to-right order of the written numbers.

However, preschool children who map spoken names to written forms with reasonable accuracy also make relative magnitude judgments when given only the written forms (Yuan et al., 2019), for example, judging "352" to be more than "235." Computational models of statistical learning (Grossberg & Repin, 2003; Yuan et al., 2020) indicate that magnitude comparison can emerge solely from knowing how multidigit number names map to written forms. In these models, the regularities across spoken name, spatial positions, and verbal markers (e.g., "hundred," the syllable "-ty") yield latent knowledge of places as markers of different magnitudes, that is, to the insight that there are places that signify quantities that decrease in magnitude from left to right in the written form. Thus, early knowledge about reading and writing multidigit numbers appears to yield approximate knowledge about places and their relative magnitudes. Finally, this early approximate and incomplete knowledge about multidigit numbers has been shown to predict later success in learning count + unit syntax in school (Mix et al., 2022).

Statement of Relevance

Learning starts early, continues for a lifetime, and takes place both inside and outside of schools. Determining how early informal knowledge is connected to later formal learning is essential to effective education. Using network analyses, we found that reading and writing numbers is a central skill that integrates pieces of knowledge about places with base-10 principles. The findings show how early imprecise knowledge about multidigit numbers—as evidenced in reading and writing numbers—sets the stage for and then is transformed by a year of in-school formal instruction. The findings have direct implications for measuring and teaching place value. The findings also have broader implications for understanding many forms of knowledge growth that are initially piecemeal but depend on principled integration for mastery.

In this study, we focused not on prediction from early knowledge to latter success but on the organization of children's early knowledge and how that early knowledge may reorganize as the result of classroom instruction. Figure 1 shows the six tasks we used and the minimal knowledge required to succeed in each task. The six tasks have been individually used in many prior studies and all correlate with other measures of early mathematics abilities (Byrge et al., 2014; Mix et al., 2014, 2016, 2017; Yuan et al., 2019). For a full list of references for the external validity of each measure predicting later mathematics achievement, see the Supplemental Material available online. We chose these tasks, however, not for their measurement properties with respect to predicting later success but because of the components of emerging knowledge that they require. Three of the tasks measured approximate knowledge about places: (a) the bidirectional mapping of multidigit number names and written forms (e.g., Byrge et al., 2014), (b) the relative magnitude of two written forms (e.g., Durand et al., 2005), and (c) the position of written numbers on a number line (Siegler & Opfer, 2003). Three additional tasks measured more precise knowledge of the counts and units that are the foundation of the syntactic principles of place value: (a) the units counted at each place (e.g., Hanich et al., 2001), (b) the quantity of any set of objects that can be determined by counting and adding together base-10 units (Chan et al., 2014), and (c) the equivalence of a multidigit number to the sum of the quantities indicated by each place (Mix et al., 2017).

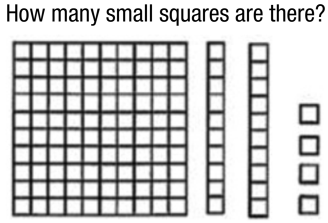
Instructions	Example trial	Knowledge required
Approximate Measures		
Reading and Writing		
Given a written number, say the name. Given a spoken name, write the number.	What is this number? 154 Write two hundred forty-seven	Knowledge required: The order of spoken digit names aligns with a left-to-right order of written digits
Magnitude Comparison		
Given two written numbers, choose the one that signifies more.	Which is more? 119 191	Knowledge required: The relative magnitudes of single digits and the relative magnitudes of places
Number-Line Estimation		
Indicate the position of a written number on a number line from 1 to 100.	Mark where you think the number at the top should go on the line 	Knowledge required: The relative magnitudes of single digits, the relative magnitudes of places, and the global magnitudes of individual multidigit numbers relative to all other numbers 1 to 100
Syntactic Measures		
Digit-Place Correspondence		
Count the units for a single place.	Which number has two thousands? 2,513 25 5,123	Knowledge required: The unit counted for each place
Base-10 Counting		
Determine quantity by counting base-10 units.	How many small squares are there? 	Knowledge required: The represented quantity is the sum of the count of units of 100, units of 10, and units of 1
Expanded Notation		
Recognize the equivalence of an expanded representation of quantity.	Which of these add up to be equal to this number? 83 800 + 3 80 + 3 8 + 3	Knowledge required: The quantity signified by a multidigit number can be decomposed into the sum of the quantities of each place

Fig. 1. Description of the six place-value tasks. For each of the three approximate measures and the three syntactic measures, the figure shows the instructions (left column), an example trial (middle column), and the minimal knowledge required to answer the question correctly (right column).

Method

This secondary data analysis used previously published data from a study that tracked performance on a range of place-value measures from kindergarten to first grade (full details of the data collection are available in the article by Mix et al., 2022). Children were tested late in the spring in kindergarten and first grade.

Participants

A total of 279 kindergartners from the United States were tested (135 females, 144 males; mean age = 5.76 years, *SD* = 0.55). One year later in first grade, 232 of these same children were retested (47 children had left the school district). Children were recruited from eight elementary schools across two different states. For the third state where data collection took place, children were recruited from the community and tested in the university's laboratory. These children were from 34 different schools. The participating communities included suburbs of a major metropolitan area on the East Coast, suburbs of a medium-size city in the Midwest, and a mixture of suburban and rural communities surrounding a small city in the Midwest. We obtained demographic data from 46% of the total sample. Not all families were given the demographic questionnaire because their schools used an opt-out consent process, and other families did not return the questionnaire after receiving it. For these families, we used school-wide information to estimate racial- and ethnic-identity distributions, and we used 2017 neighborhood census data to estimate median income. Weighted descriptive statistics indicated that the sample was racially diverse (42% Black, 37% White, 8% Asian, 13% Latino) and primarily of middle socioeconomic status (average median family-income range = $75,000 to $99,999).

The large sample from many different kinds of communities was expected to tap the variety of skills that children bring to formal schooling and the variety of classroom approaches to teaching place value. Given the state education guidelines and Common Core State Standards, place-value instruction does not begin until first grade, although curricula in kindergarten often include counting to 100 and introduction to the teens (National Governors Association Center for Best Practices, Council of Chief State School Officers, 2010). However, the curricula of participating classrooms were not directly measured and likely varied. Preschool experiences were also likely variable.

A sensitivity test was conducted in G*Power (Version 3.1; Faul et al., 2009) before data collection for the larger study to determine what sample size would be adequate to detect a medium effect based on regression models (Mix et al., 2022). The results indicated that a sample size of 120 was adequate to detect a medium effect (i.e., Cohen's f^2 = .11; Cohen, 1988) with the following parameters: an α of .05, power of .80, a sample size of 120, and six predictors. Consequently, the sample had more than adequate power for the larger study. The stability of the network's centrality indices was analyzed (for further detail, see the Analysis Plan section).

Procedure and materials

Testing sessions for most children took place in a quiet area outside the classroom; some children were tested in the laboratories of the participating researchers. Testing lasted approximately 60 min per child. The six tasks (Fig. 1) were administered individually in two random orders, counterbalanced across children. Reliabilities were calculated at both time points using Cronbach's α.

Approximate measures. In the *reading and writing* task—hereinafter referred to as "*reading/writing*" (e.g., Byrge et al., 2014)—children bidirectionally mapped spoken multidigit names and written notation. On reading trials, children saw a stimulus number (e.g., "423") and said its name aloud (e.g., "four hundred and twenty-three"). If the child said the entire name correctly, the trial received a score of 1. If any part of the name was incorrect, the trial received a score of 0. On writing trials, children wrote numerals after being given the spoken name. If the child wrote the entire numeral correctly, the trial received a score of 1. If any part of the numeral was incorrect, the trial received a score of 0. The 12 test trials probed two-, three-, and four-digit numbers (maximum score = 12; kindergarten: α = .87, first grade: α = .87).

In the *magnitude-comparison task* (Mix et al., 2014), children were asked which of two written numerals represented the larger quantity (e.g., 461 vs. 614). There were 25 trials composed of one- to four-digit numerals (maximum score = 25, chance = 12.5; kindergarten: α = .72, first grade: α = .79).

In the *number-line-estimation task* (Siegler & Opfer, 2003), children were presented with a blank number line ranging from 0 to 100 and were told to indicate where a number (e.g., 36) should be located using a vertical hash mark. Following one practice trial with feedback, children completed 15 test trials. We coded test trials for percentage of absolute error by measuring the distance from the hash mark made by the child to the correct location and then dividing by the scale of the number line (i.e., 100; Booth & Siegler, 2006). Scores were then transformed into percentages, and each score was subtracted from 100%, so that higher scores indicated better performance. The total score was the percentage of accurate responses averaged across the 15 test trials (range = 0–100%, even-odd reliability r at kindergarten = .76 and at first grade = .74).

Syntactic measures. The *digit-place-correspondence task* was a multiple-choice adaptation of the digit-correspondence task (e.g., Hanich et al., 2001). Children were shown three written numerals (e.g., 2, 20, and 10) and asked a place-value question such as, "Which number has two tens?" The six test trials probed tens, hundreds, and thousands (maximum score = 6, chance = 2; kindergarten: α = .53, first grade: α = .59).

In the *base-10 counting task* (Chan et al., 2014), children were asked to determine the quantity indicated by line drawings of base-10 blocks that represented two- and three-digit numbers (see Fig. 1). Counting large quantities by counting the individual "ones" units (e.g., 1, 2, 3, . . . , 89, 90, 91, 92) not only is inefficient but also leads to errors (e.g., skipping a unit). However, counting the base-10 units and then correctly combining them readily leads to accurate determination of the exact quantity represented (e.g., 10, 20, 30, . . ., 80, 90, 91, 92). The 10 trials were coded as correct if the count, however achieved, matched the correct total (maximum score = 10; kindergarten: α = .85, first grade: α = .82).

In the *expanded-notation task* (e.g., as used by Mix et al., 2017), children were shown a written numeral (e.g., 11) and asked to select the correct expanded version from among three options (e.g., 10 + 1, 10 + 10, or 1 + 1). The six test trials probed two-digit, three-digit, and four-digit numbers (maximum score = 6, chance = 2; kindergarten: α = .66, first grade: α = .70).

Analysis plan

Because our goal was to describe the structure of knowledge at the two points in learning, networks of associations among the six tasks were created separately for kindergarten and first-grade performance. The six nodes of each network represented the tasks, and edges represented statistical associations between performance on those tasks (Epskamp et al., 2018). Because the data were continuous and followed a multivariate normal density, the appropriate pairwise Markov random-field model was the Gaussian graphical model (Epskamp et al., 2018). A Gaussian graphical model estimates the network structure by computing edge weights that reflect partial Pearson correlation coefficients between two nodes after controlling for all other variables in the network. Figure 2 illustrates our analysis approach using three hypothetical networks.

We used two global connectivity measures. The first is *network density*, which is the number of observed edges relative to the total number of possible (nondirectional) edges (e.g., 15 total possible edges for a

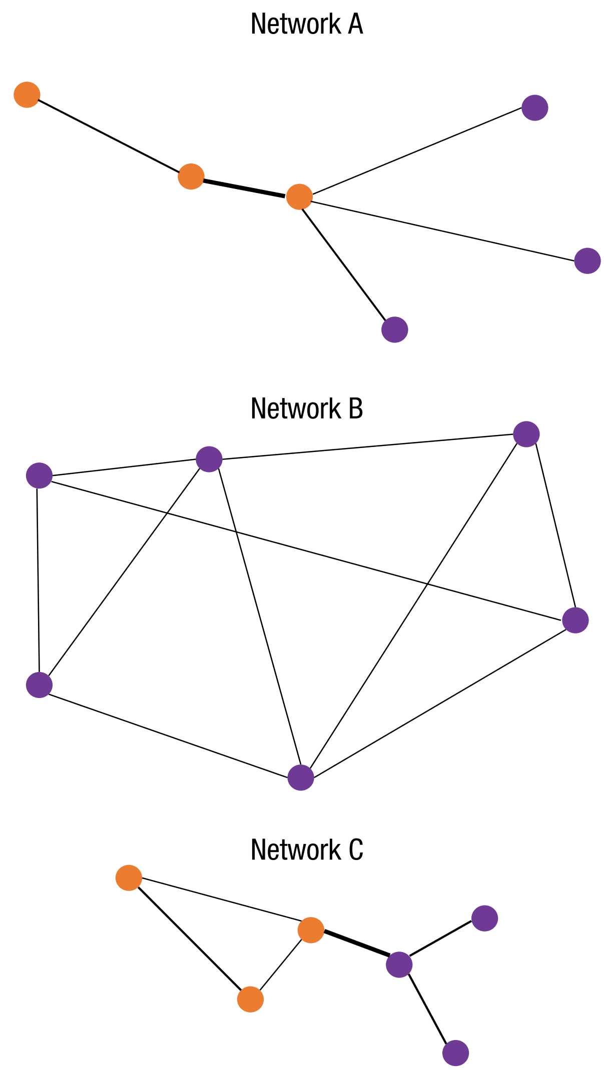


Fig. 2. Example network visualizations. Edge strength is illustrated by the thickness of the edges (i.e., lines connecting each node); thicker edges indicate greater edge strength. These example network visualizations demonstrate a few of the possible structures that could be estimated in both the early and later knowledge networks. First, one possible change is that early and later knowledge systems could differ in connectivity: Early knowledge, for example, could be more piecemeal or diffuse, with the components less strongly connected to each other (network A), and later knowledge could be more integrated and closely connected (network C). Another possibility is that approximate and syntactic knowledge could form separate clusters of knowledge characterizing both early and later knowledge systems (networks A and C). Alternatively, knowledge across tasks—at one point in learning or the other—could be more equal contributors to a single system (network B). The knowledge systems could also be characterized by central or "hub" nodes (Baronchelli et al., 2013) that connect all the components of knowledge (e.g., networks A and C with two center nodes). A central hub node implies a focal component that is critical to integrating other components into a system and in the present case would implicate a component of knowledge that may be needed to hold the whole knowledge system together. Evidence for a developmentally stable hub connecting approximate and syntactic knowledge would have not only important theoretical implications for the emergence and integration of a coherent knowledge structure of multidigit understanding but also practical implications for informal and formal educational experiences.

six-node network). A network in which every node is directly connected to every other node is more connected but less structured than one in which only some of the possible paths between nodes are realized. The second measure is *average absolute edge strength*, which is the mean network weight of all estimated edge weights (Burger et al., 2022). A highly structured network might be expected to have few edges that are strong and thus a high mean edge strength. In Figure 2, network A has weak global connectivity: There are relatively few edges, and they are not strong. Network B is highly connected, Individual nodes have connections to many other nodes, but the individual edges are generally weak. Network C is strongly connected but in a different way; there are fewer connections, and some edges are much stronger than others. Network B would indicate strong connections among all the tasks. Network C would implicate a potentially more interesting structural pattern in which some nodes may be more important than others in controlling the relations in the network.

Centrality measures are used to determine whether some nodes are more central than others (serving as "hubs" in the paths among nodes; Baronchelli et al., 2013). A central hub node implies a focal component that is critical to integrating other components into a system and in the present case would implicate a component of knowledge that may be needed to hold the whole knowledge system together. We previously hypothesized that reading and writing multidigit numbers is a key entry point for approximate knowledge and might be expected to be the central node before instruction (Yuan et al., 2020). If instruction transforms that knowledge, the first-grade network could show a different structure, in which tasks involving count + unit syntax perhaps are more central.

To measure these possibilities, we used three centrality indices: *node strength*, *node closeness*, and *node betweenness*. Node strength estimates how tightly individual nodes are directly connected to other nodes by computing the sum of absolute partial-correlation coefficients (i.e., path lengths) between each node and all other nodes. A node that is strongly connected to many other nodes is considered more central. Node closeness estimates connection strength as the inverse of the sum of the shortest path lengths from each node to every other node. The shorter the path from a node to all other nodes, the stronger the associations of that node to all other nodes. Node betweenness measures the number of times each node connects two other nodes as measured by the shortest path between those nodes (Epskamp et al., 2018). Nodes that participate in more of the shortest paths between other nodes play a stronger role in the relational structure. In the three examples in Figure 2, networks A and C have two nodes that are more central than other nodes in the network: These two nodes are more strongly connected than other nodes, they are connected by shorter paths to other nodes, and they must be travelled through for other nodes to connect to each other. By hypothesis, reading and writing numbers may be an early skill that is central to connecting emerging abilities; if so, the node representing reading and writing multidigit numbers should be more central to performance in other tasks in kindergarten. However, this hub structure could change after first-grade classroom instruction in count + unit syntax.

To determine the stability of the centrality indices, we used case-dropping subset bootstrapping that reestimated the network with smaller sample sizes and quantified the stability of the indices using *correlation-stability* (CS) *coefficients* with 1,000 bootstrap iterations (see the Appendix). We interpreted only the results for CS coefficients that exceeded a .50 threshold (Epskamp et al., 2018). We assessed the accuracy of the estimated edge weights using 95% bootstrapped confidence intervals (CIs) for both node strength and edge-weight strength, and we compared relative weights using the bootstrapped difference test in the *bootnet* R package (Version 1.5; Epskamp et al., 2018) with an α of .05 based on 2,500 bootstrap iterations. Missing data were excluded from the network analysis, including using pairwise deletion for the network-comparison test.

Cluster measures determined whether the tasks partitioned into separate constructs. For example, early preinstruction knowledge could have divided into two categories aligning with our prior distinction between approximate and syntactic knowledge, but these clusters could have reorganized or disappeared if formal instruction transformed the whole system of knowledge rather than merely adding new content. For the cluster analyses, we used the spinglass community-detection algorithm. This algorithm assumes that edges should connect nodes of the same spin state (i.e., community) and that nodes with different states or communities should be disconnected. The *igraph* spinglass algorithm was selected because it is well suited to smaller networks (Yang et al., 2016). We conducted 1,000 iterations and reported the median number of clusters.

Finally, for the analyses and data visualization, we used the conservative "least absolute shrinkage and selection operator" (LASSO; Tibshirani, 1996), suitable for networks with a small number of nodes and edges. We used the *qgraph* package in R (Version 1.9; Epskamp et al., 2012) and the graphical LASSO (*glasso*; Friedman et al., 2008) algorithm to estimate a range of networks, after which we used LASSO regularization (using a tuning-parameter γ set to 0.5) to select a network with

Table 1. Change in Task Performance From Kindergarten to First Grade

Task	Kindergarten *M* (*SE*)	First grade *M* (*SE*)	Difference	Paired-samples *t*	*d*
Reading and writing	−0.41 (0.06)	0.49 (0.06)	0.89 [0.79, 0.99]	$t(230) = 17.62$, $p < .001$	1.01
Magnitude comparison	−0.41 (0.06)	0.49 (0.05)	0.90 [0.78, 1.02]	$t(227) = 14.65$, $p < .001$	1.02
Number-line estimation	−0.34 (0.06)	0.41 (0.06)	0.74 [0.63, 0.86]	$t(230) = 12.75$, $p < .001$	0.81
Digit-place correspondence	−0.15 (0.06)	0.18 (0.07)	0.35 [0.20, 0.50]	$t(227) = 4.66$, $p < .001$	0.33
Base-10 counting	−0.40 (0.05)	0.48 (0.06)	0.89 [0.79, 1.00]	$t(230) = 16.66$, $p < .001$	0.98
Expanded notation	−0.44 (0.06)	0.52 (0.05)	0.98 [0.86, 1.11]	$t(224) = 15.33$, $p < .001$	1.10

Note: Values in brackets are 95% confidence intervals. For density plots, see Figure 3.

the best model fit. We used multidimensional scaling (MDS) with the *smacof* R package (Version 2.1-1; de Leeuw & Mair, 2009) to visualize the estimated networks (see Jones et al., 2018, for a tutorial), representing nodes with stronger connections as closer in the visualization. The accuracy of the MDS layout was assessed using reported stress-1 values of the MDS fit (Mair et al., 2016).

Results

We first report on children's accuracies in the six tasks and their improvement from kindergarten to first grade. We then turn to the main questions about the organization of children's knowledge. Because the six tasks have very different measurement properties (number of trials, magnitude of the numbers queried, and chance), we computed task-specific *z* scores combining the kindergarten and first-grade performances. In this way, we measured degree of growth (Table 1) from kindergarten to the end of first grade relative to the distributions of performances in each task across the two testing times. As shown in Figure 3, density plots illustrate that the distributions consistently move rightward, showing increased accuracy on all tasks from kindergarten to first grade. The three approximate tasks all show a broad range of performances at the first test, but in first grade, there was less variation, with the mode near the top of the range of performance for all children. However, a significant number of first-grade children's scores still fell below the mean of kindergarten performance on these tasks. Indeed, scores of 68 first graders (29%) fell below the overall mean performance (indicated by 0 on the *x*-axis) in the hypothesized entry task of reading and writing multidigit numbers.

Performance on the syntactic tasks showed a similar overall pattern, with a broad range of performance in kindergarten but with many children performing similarly to kindergarten level after a year of instruction in first grade. This descriptive analysis of performance yields two critical points relative to interpretation of the network analyses: (a) Instruction is associated with improved performance in all tasks, with the mode in first grade approaching the high-performance limit, and (b) individual differences are nonetheless considerable at both grade levels, with children's performance at each grade level spanning nearly the entire distribution of performance for the two grades combined.

Kindergarten network

We used a common visualization approach (Epskamp et al., 2012) in Figure 4 (top) that revealed the structure of component knowledge by successively removing weaker edges. The best-fitting network had an estimated mean edge weight across all possible edges of .15 distributed over 13 of the 15 possible edges. Thus, overall, the network was diffusely connected with many weak edges, on average. However, some edges were much weaker than others. This was easily seen by removing all edges with strengths below .2. Using this imposed threshold, we found that only five tasks were connected via four edges forming a string structure of tasks. If we removed the weakest links in this string, imposing an edge-strength threshold of .4 for visualization of an edge, only two tasks remained—reading/writing and base-10 counting—with one edge connecting them. Thus, the strongest regularized, partial-correlation link was between the hypothesized entry knowledge of reading and writing multidigit numbers

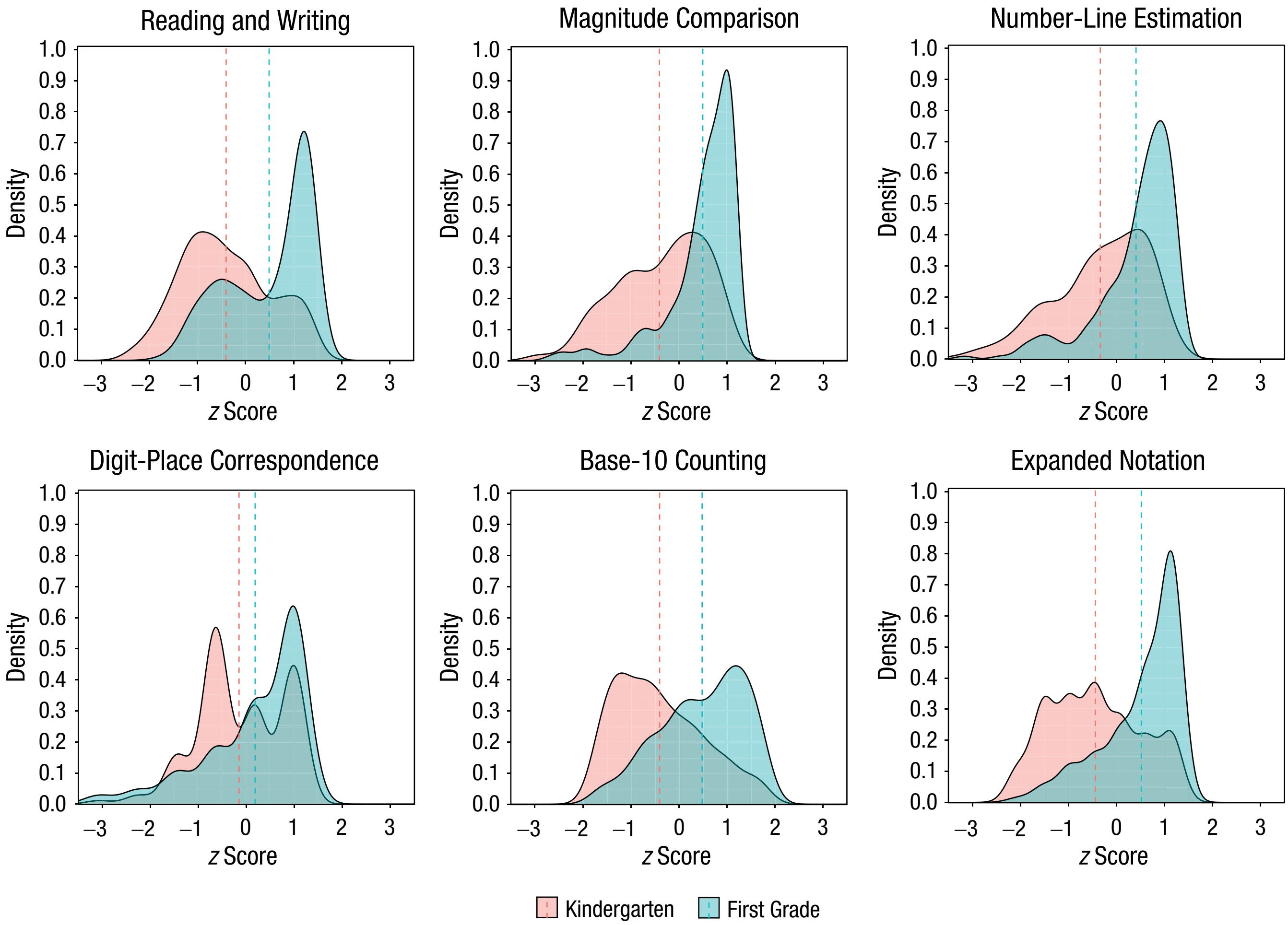


Fig. 3. Density plots of scores on each of the approximate measures (top row) and syntactic measures (bottom row), separately at kindergarten and first grade. Values are *z* scored for easier comparison. Dashed vertical lines indicate group means. The overall mean is indicated by 0.

and, unexpectedly, a measure of count + unit syntax that asked children to determine the quantity of large numbers (and offered base-10 physical groupings as a means of doing so).

The planned centrality measures confirmed that knowing how number names and written numbers map to each other (reading/writing) and determining the exact quantity of large (and not easily countable by ones) amounts (base-10 counting) are central knowledge hubs within the kindergarten network. These two tasks had the highest node strengths, indicating stronger connections to all of the other nodes (1.10 for base-10 counting and 1.00 for reading/writing; see Fig. 5), but the strengths did not differ from each other (95% CI = [−.388, .206], bootstrapped difference tests; see Fig. 6). Node closeness of both reading/writing (.04) and base-10 counting (.04) were significantly higher than the four other nodes and did not significantly differ from one another (95% CI = [−.009, .008], bootstrapped difference test). Finally, the edge weight between these two central nodes (edge weight = .40; see Figs. 6 and 7) was significantly higher than for most other edges except for the base-10 counting and expanded-notation edges (edge weight = .32; 95% CI = [−.230, .049])—the second-strongest edge.

As shown in Figure 4, the cluster analysis identified two communities in the stringlike structure that aligned with our a priori categories of approximate and syntactic tasks. However, the most notable finding was a diffuse network with just two strongly connected hub abilities: (a) reading/writing multidigit numbers, the entry and imperfect knowledge of how spoken number names map to written forms, and (b) base-10 counting, a task that uses base-10 units to determine the quantity of a large set of discrete entities and thus requires at least some knowledge of places that represent base-10 units and how these combine to represent the entire quantity.

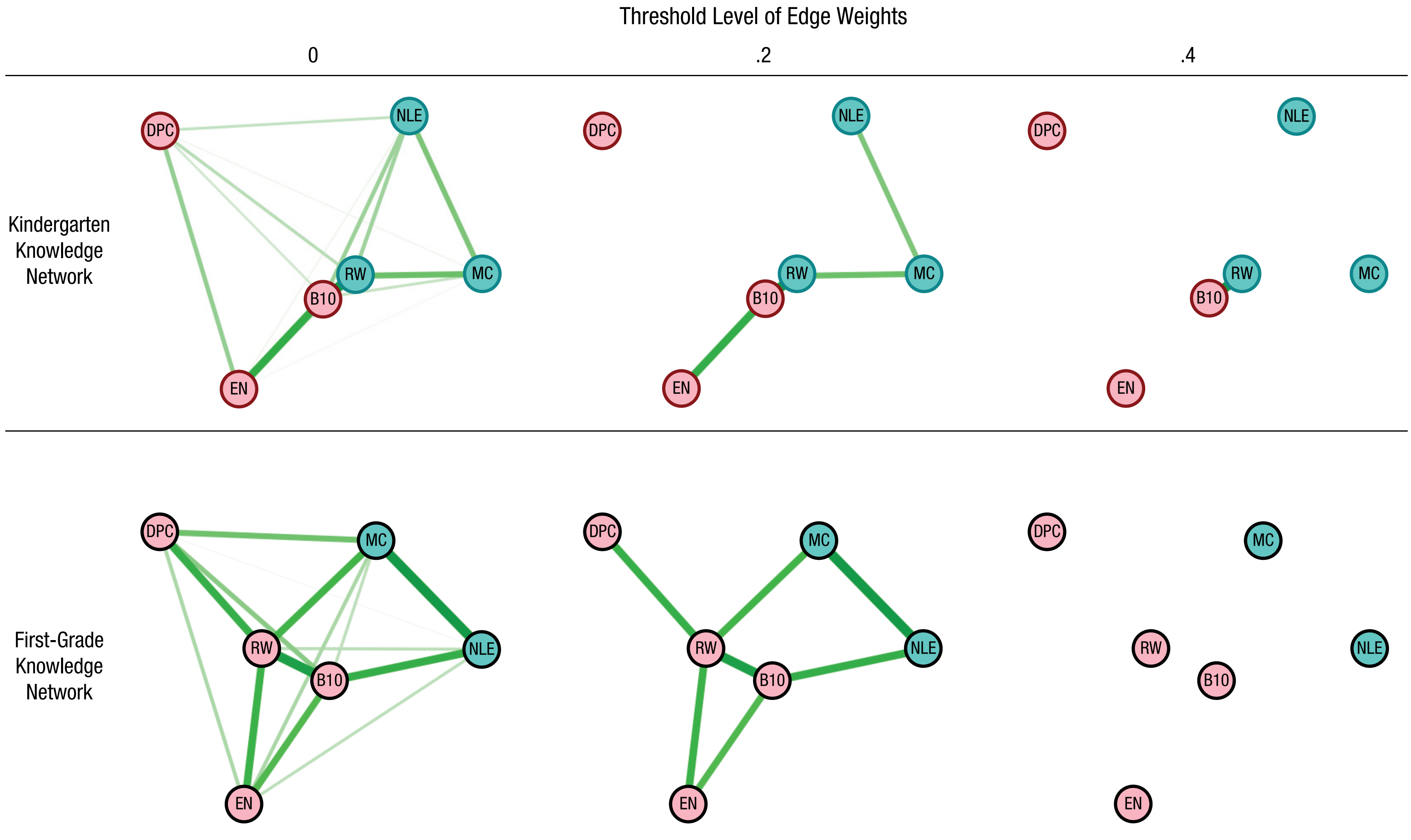


Fig. 4. Estimated network structures of the six place-value tasks across varying levels of edge-weight thresholds, separately for kindergarten knowledge and first-grade knowledge. The six nodes (i.e., place-value tasks) are represented by circles, and the edges are represented as lines. All edge weights are positive. Thicker lines indicate stronger edge weights or partial-correlation coefficients. A threshold of 0 indicates no threshold, so all estimated pathways are displayed. A threshold of .2 allows estimated edge weights that are above .2 to be shown, whereas a threshold of .4 allows edge weights above a .4 threshold to be shown. Multidimensional scaling was used to visualize the network edges estimated from the graphical "least absolute shrinkage and selection operator" (LASSO) network but with spacing based on zero-order correlations. Consequently, nodes that are close together are similar in terms of zero-order correlations, but nodes that share thick edges are similar in terms of regularized partial correlations. The border color of the node represents the predicted network cluster (i.e., a dark red border illustrates predicted count + unit syntax tasks; a dark turquoise border illustrates predicted approximate tasks). The fill color of the node represents the resulting estimated network clusters from the cluster analysis (i.e., the pink fill illustrates resulting count + unit tasks; the turquoise fill illustrates resulting approximate tasks). For the first-grade knowledge network, the border color of all nodes is black because we expected the nodes to be more integrated and not form clusters; however, the cluster analysis yielded two clusters as illustrated by the fill colors. B10 = base-10 counting; EN = expanded notation; DPC = digit-place correspondence; NLE = number-line estimation; RW = reading/writing; MC = magnitude comparison.

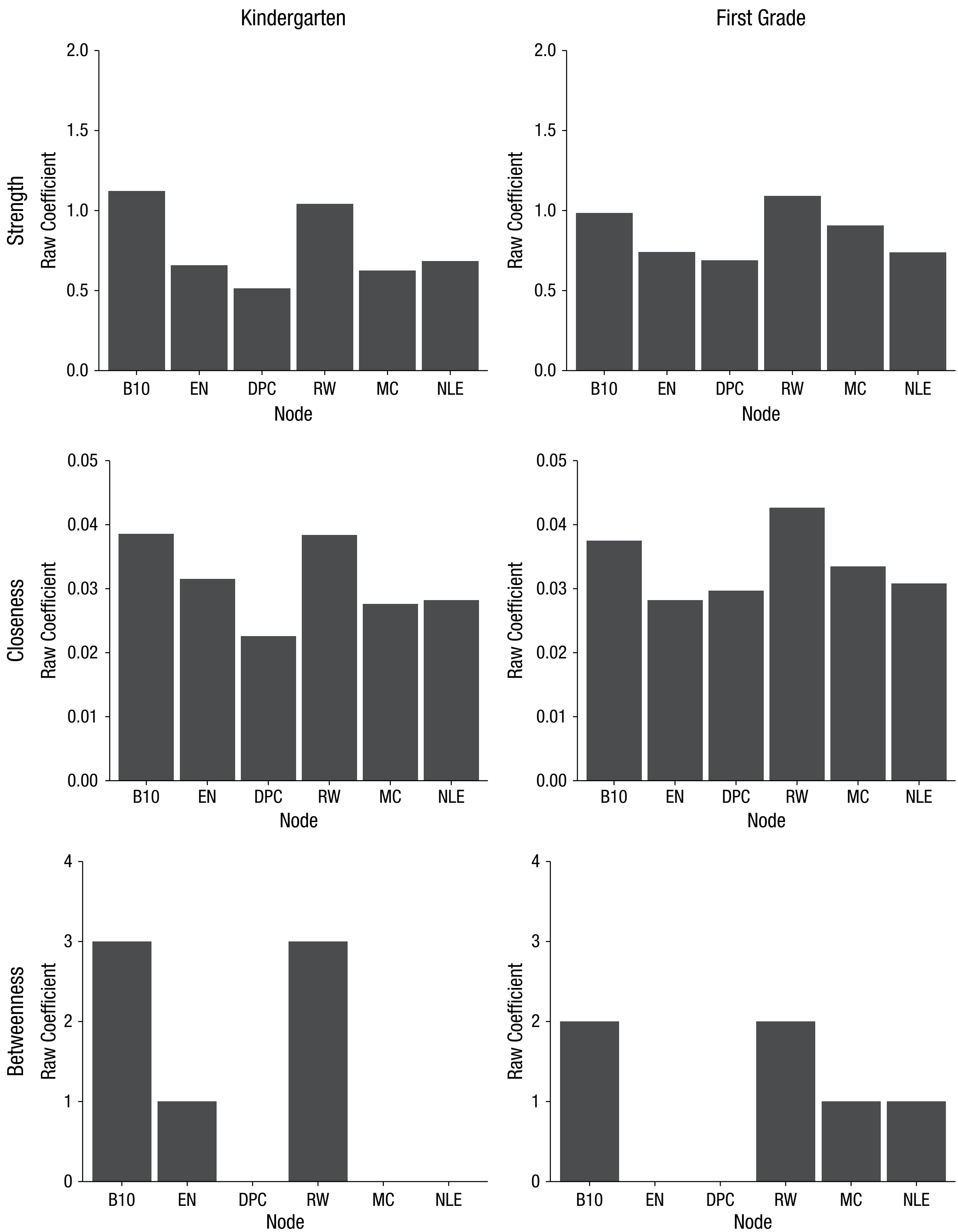


Fig. 5. Early (kindergarten) knowledge and later (first-grade) knowledge centrality indices for the estimated network of the six nodes. Results are shown separately for each centrality index (node strength, node closeness, and node betweenness). Raw coefficients are shown on the *y*-axes. B10 = base-10 counting; EN = expanded notation; DPC = digit-place correspondence; RW = reading/writing; MC = magnitude comparison; NLE = number-line estimation.

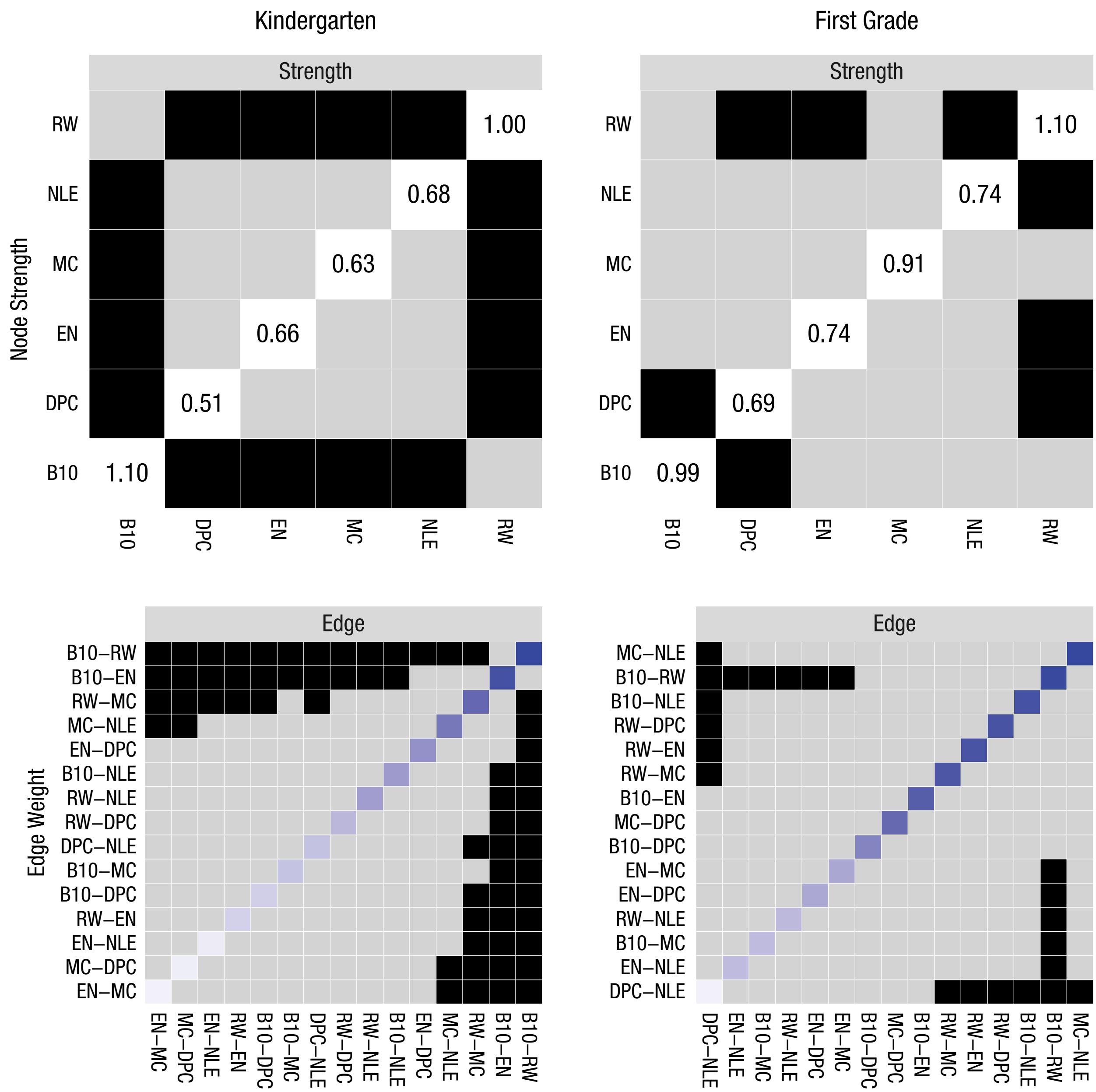


Fig. 6. Results of bootstrapped difference tests (α = .05) for both the early (kindergarten) and later (first-grade) networks. The top row shows differences between node strengths of the six tasks; the bottom row illustrates edge weights in the estimated networks. Black boxes in the matrices represent nodes (top row) or edges (bottom row) that significantly differ from one another; gray boxes represent nodes or edges that do not significantly differ from one another. White boxes in the strength-centrality plots (top row) show the mean value of the node's strength (e.g., the base-10 [B10] node strength for kindergartners is 1.10). The saturation of the blue-colored boxes in the edge plot (bottom row) illustrate the relative strengths of the edge weights (darker shades indicate stronger edge weights). EN = expanded notation; DPC = digit-place correspondence; RW = reading/writing; MC = magnitude comparison; NLE = number-line estimation.

First-grade network

Figure 4 (bottom) represents the structure of knowledge after classroom instruction in first grade. The best-fitting network had an estimated mean edge weight (distributed across all possible 15 edges) of .17. When weaker edges (with strengths less than .20) were removed in the visualization, there was a clear structure of interconnection across the six tasks, with reading/writing numbers and base-10 counting still forming a hub. However, the even distribution of connectivity among tasks was evident when the threshold for visualizing an edge is raised to .40; at this threshold, all edges disappear. As in the kindergarten network, reading/

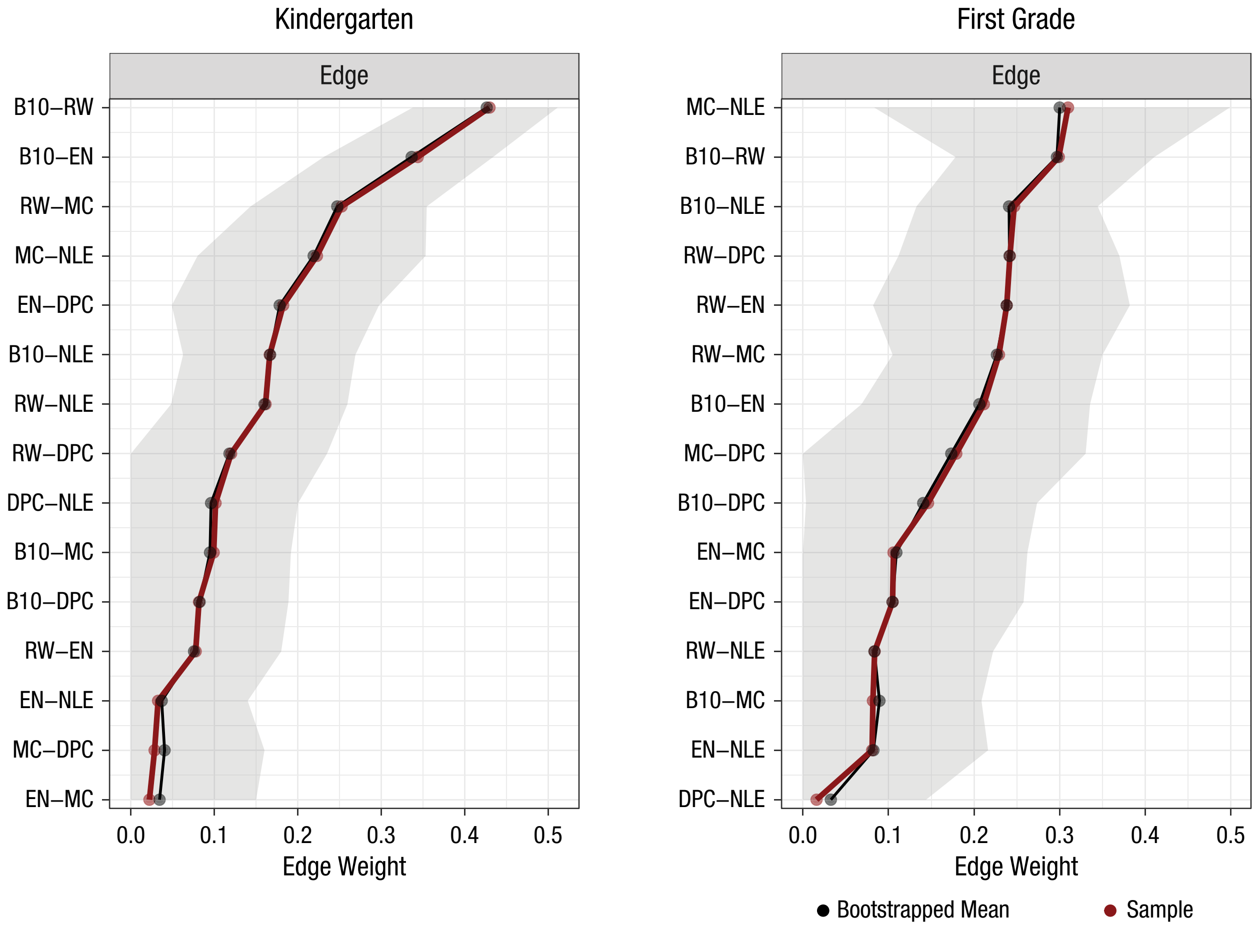


Fig. 7. Bootstrapped 95% confidence intervals of estimated edge weights for both the early (kindergarten) knowledge and later (first-grade) knowledge six-node estimated networks. Confidence intervals are represented by the gray area: Each edge is labeled along the *y*-axis. The mean edge weights of the sample are represented by the red line, and the bootstrapped mean is represented by the black line. B10 = base-10 counting; EN = expanded notation; DPC = digit-place correspondence; RW = reading/writing; MC = magnitude comparison; NLE = number-line estimation.

writing and base-10 counting had the highest node strengths (1.10 and .99, respectively) and did not significantly differ from each other (95% CI = [–.167, .409], bootstrapped difference test). They also did not differ from magnitude comparison (strength = 0.91), a task that measures understanding of the way places represent relative magnitudes. The reading/writing task's node strength was significantly greater than the node strength for expanded notation, number-line estimation, and digit-place correspondence (see Fig. 6), indicating that this hypothesized entry knowledge plays an organizing role in the first-grade network. The edge weight between reading/writing numbers and base-10 counting was significantly greater than the other six edges (see Figs. 6 and 7), indicating the continued centrality of this connection within the larger network. Because of low CS coefficients of node closeness and betweenness (see Table 2), these indices will not be discussed or interpreted. Finally, the cluster analysis revealed a notable shift in communities of knowledge: The spinglass community detection located reading/writing with the syntactic tasks that measure precision-level knowledge of the units being counted at each place, separate from the two remaining approximate tasks—magnitude comparison and number-line estimation—that require only relative knowledge of how places differ in the magnitudes indicated.

Network change

We assessed changes in knowledge structure from kindergarten to first grade in (a) global network structure, (b) global network strength (i.e., weighted absolute sum of all edges), and (c) specific edge weights between the early- and later-knowledge networks using the network comparison test—a two-tailed permutation test—for repeated paired measurements with 1,000 iterations (van Borkulo, 2019). The network comparison test revealed that although the global network structure did not change across time points, p = .445, there was a significant increase in global network density from kindergarten (strength = 2.31) to first grade (strength = 2.57), S = 0.26, p = .001. This global effect was primarily due to the increase in the edge weight between reading/writing numbers and expanded notation, which is the only edge with a significant weight increase from kindergarten to first grade, p = .031. This stronger connection of reading/writing numbers to a syntactic task is consistent with the node's overall shift toward the syntactic tasks revealed in the cluster analysis.

A visual comparison of the networks at different threshold levels (Fig. 4) indicates distinct structures at the .20 threshold—a level close to the mean edge weight of the first-grade network. Specifically, the number of estimated edges was higher in the first-grade network (seven) than it was in the kindergarten network (four). The degree of centrality (i.e., the number of connections with other nodes) of reading/writing numbers was twice as high in the first-grade network (four) as it was in the kindergarten network (two). Additionally, reading/writing had a higher node strength (1.01) and closeness (23.44) in the first-grade network compared with the kindergarten network (0.68 and 19.97, respectively). Both metrics suggest that the strength of the reading/writing edges as well as the global access of nodes with reading/writing are stronger in the first-grade network, confirming the strong hold that the reading and writing of multidigit numbers has in the acquisition of place-value concepts from kindergarten to first grade.

Table 2. Network-Analysis Accuracy Metrics for Node Central Stability

Network	Strength	Closeness	Betweenness
Kindergarten	.67	.67	.28
First grade	.52	.28	.00

Note: Correlation-stability coefficients are displayed for each of the three measures of node central stability. Values below .50 are considered low, and interpretation should be done lightly, whereas values below .25 should not be interpreted (Epskamp et al., 2018).

Discussion

The main findings are these. Early knowledge about multidigit numbers is piecemeal. Performance in different tasks is visually organized as a string with approximate tasks on one side and count + unit tasks on the other, and with both clusters connected in the middle by a strong link between reading and writing multidigit numbers and base-10 counting. After a year of in-school instruction, the string was folded in on itself. Performance in each task was more strongly connected to every other task, with reading/writing and base-10 counting forming a central hub. After instruction, reading/writing was more strongly associated with tasks requiring knowledge of count + unit syntax tasks than with tasks measuring knowledge of relative magnitudes. The overall pattern implicates two hub skills at the center of both pre- and early post-first-grade-instruction learning. The pattern also indicates a transformation in one of these hubs after instruction. We hypothesize that reading/writing becomes more strongly linked to the count + unit tasks because, with instruction, children interpret the reading/writing task explicitly in terms of counts of base-10 units rather than relying on piecemeal understanding and heuristics.

Although many children prior to formal instruction perform well above chance on reading and writing multidigit numbers, they also make errors—with transpositions when zeros are present—that indicate a lack of understanding of the principles underlying base-10 notation (e.g., Byrge et al., 2014). Some older children after multiple years of instruction make these same errors, and these children have difficulties in performing multidigit calculation (e.g., Cooper & Tomayko, 2011). These facts have led to discussions within the education literature about when and how to introduce children to multidigit numbers and specifically whether multidigit numbers should be introduced in the context of explicit instruction about count + unit syntax (e.g., Baroody, 1990; Fuson, 1990; McGuire & Kinzie, 2013). The current results contribute to these discussions, suggesting that early reading and writing of multidigit numbers, albeit imperfect, is nonetheless an entry point to base-10 principles. The difficulties of older children who still make preschool-like errors may be a signal that these children's approximate understanding of how the symbols (i.e., number names and written forms) work was not transformed by formal instruction. Because the relational structure of spoken number names and written forms varies across languages (Ho & Fuson, 1998), an important open question is whether the reorganizational patterns are language dependent and differ across different language communities.

Studies of statistical learning show that imperfect regularities in everyday language experiences lead to latent implicit knowledge of abstract syntactic categories of nouns and verbs (e.g., Colunga & Smith, 2005; Wells et al., 2009). One recent training study (Yuan et al., 2020) showed that preschool children could acquire generalized knowledge of how spoken multidigit number names map to written forms, given training on a very sparse sampling of name-form pairs from 1 to 1,000. Computational models (Grossberg & Repin, 2003; Yuan et al., 2020) have shown that learning of these correspondences between names and written forms also leads to latent approximate knowledge of places. We conjecture that this early latent knowledge constitutes an approximate anchor for learning explicit principles. Nouns in language, for example, do not refer to the common-sense notion of "people, places, and things" but are formally defined by the roles of words in sentences and the transformations that relate one sentence to another. So too, places do not refer to different-sized units but are formally defined by the base-10 multiplicative hierarchy that relates the component symbols within a represented number and the transformations that relate one multidigit number to any other. The explicit understanding of the syntax and the computational use of it—in both linguistics and mathematics—appears to require formal instruction. The early, not-quite-right latent notions that are difficult to verbalize may be the internal anchors on which the success of formal instruction depends. From this perspective, the centrality of reading and writing multidigit numbers (i.e., mapping names with written forms) makes sense. Before formal instruction, encounters with the names and written forms provide the statistics from which the intuitive anchors emerge (Yuan et al., 2019).

Two hypotheses suggest themselves as to why some children still struggle to master place-value principles late in elementary school. First, these children may begin formal instruction with weak intuitive knowledge and thus lack the approximate semantic anchors on which successful instruction depends. Second, different properties of instruction may connect better with the hypothesized latent concepts. For example, instruction that explicitly reminds children of what they already know about the latent structure of number names and their written forms (perhaps skipping over the teens to focus on the regularities) may benefit learning. The centrality of reading/writing and base-10 counting also suggests the potential value of instruction that explicitly links reading, writing, and counting physical models of base-10 units. These are critical questions for future research.

Much research in education is focused on finding early predictors of later learning success, the idea being that strong predictors are likely also strong players in causation. However, considerable biomedical research has made clear that prediction accuracy, validity, and intervenable roles in causal pathways are often at best complexly related (e.g., Hussein et al., 2018; Pearl, 2019). Accordingly, we sought an approach different from and complementary to prediction, one focused on specific components of early and later knowledge and their role in creating an organized and multicomponent system of knowledge about place value. Expertise in mathematics and science is built incrementally: Early not-quite-right ideas may be essential steps on a path to full understanding. The question of how early incomplete components create the context for later learning and are transformed by education into explicit and exploitable principles is central to human cognitive achievement. Place value, one of the earliest sets of mathematic principles that children learn, is a rich and useful domain in which to understand processes of knowledge growth and change.

Appendix

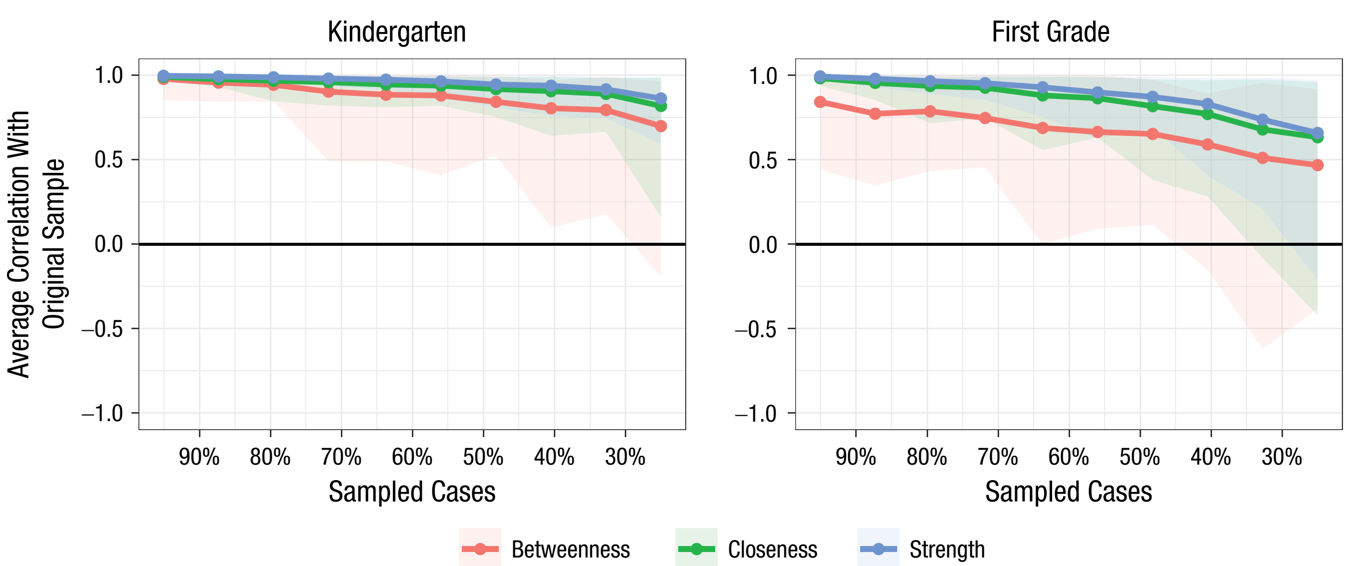


Fig. A1. Results of the case-dropping bootstrap analysis for both the early (kindergarten) and later (first-grade) knowledge networks. Each graph shows average correlations between estimated networks sampled with cases dropped and the original sample, separately for each of the three centrality indices (node betweenness, node closeness, and node strength). Lines indicate means, and the shaded subareas indicate the range from the 2.5th quantile to the 97.5th quantile.

Transparency

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Author Contributions

K. S. Mix and L. B. Smith developed the study concept. C. A. Bower planned, executed, and interpreted the network analyses. C. A. Bower, K. S. Mix, and L. B. Smith wrote the manuscript. L. Yuan provided critical insights on all aspects of analyses and writing. All authors approved the final version of the manuscript for submission.

Declaration of Conflicting Interests

The author(s) declared that there were no conflicts of interest with respect to the authorship or the publication of this article.

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Open Practices

Deidentified data and analysis scripts have been made publicly available via the Digital Repository at the University of Maryland and can be accessed at http://hdl.handle.net/1903/21495; access to the data is limited to qualified researchers. The materials used in these studies are widely available. The design and analysis plans for the study were not preregistered. This article has received the badge for Open Data. More information about the Open Practices badges can be found at http://www.psychologicalscience.org/publications/badges.

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Supplemental Material

Additional supporting information can be found at http://journals.sagepub.com/doi/suppl/10.1177/09567976211070242

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